Research Paper

Presentation of a Mathematical Model for Optimal Portfolio in the form of a Dynamic Stochastic General Equilibrium Model for Economy of Iran

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ABSTRACT
One of the most important aspects of investment is determining the “optimal investment portfolio”. To date, scientific research has been conducted to determine the optimal portfolio with “artificial intelligence” and “Fuzzy logic”. However, we determine the optimal portfolio based on Dynamic Stochastic General Equilibrium (DSGE) model. On the other hand, several factors affect returns, which is one of the most important issues in investment decision-making, and various models have been developed to analyze the return of “capital” and “other assets”. In the present study, we design and calibrate a new Keynesian DSGE model in relation to the optimal investment portfolio and effectiveness of shocks on macroeconomics variables. To this end, we design a DSGE model in the sector of household and firms, government and the central bank, and calibrate the model’s parameters after logarithm-linearization using seasonal information of 1996-2016 and results of empirical studies. In the designed model, households are divided into two groups of savers and borrowers, both of which consume, provide labor, and invest with their savings, and hold a portfolio of stocks, cash, securities and other assets based on risk, return or optimal portfolio). In the end, we assess the instantaneous reaction function of economic variables to productivity and foreign exchange earnings shocks. Ultimately, the comparison of the present torques in the current study and torques of real data in the economy of Iran is indicative of the relative success of the model in the realities of Iranian economy.

1 Introduction
After the criticism of the macro econometrics model by Lucas in 1980, dynamic stochastic general equilibrium (DSGE) models have been extensively used in the form of a school of real business cycle, and then, in the form of school of Neo-Keynesian economics. The first generation of these models evaluates the models designed for various economies by comparing them with real variables with an
emphasis on product market equilibrium and assessing the effect of various shocks on the studied variables. The second generation of DSGE models emphasizes the work market equilibrium in addition to product market, attempting to enter the Labor market variables. Ultimately, the third generation of these models deals with entering the financial market. National studies on DSGE have mainly evaluated the use of these models for Iran’s economy with an emphasis on product market. The “Money Growth Rate” is used as a replacement of the Taylor rule based on interest rate flexibility, which is a monetary policy tool not applicable in Iran’s economy, for monetary shock. Similar to other studies in the DSGE field, despite the assumption of exclusive competition in the model, the monetary shock is only able to affect costs and inflation and has no impact on real variables of the model due to lack of adhesion. In estimation of stability of various shocks, the Augmented Dicky Fuller is first used to investigate the existence of a single root, followed by the use of Hodrick–Prescott filter to divide the cycle section and the variable trend and estimate it as AR (1) process. Moreover, the Solow residual equation is exploited to determine the productivity stability coefficient. Contrary to macro econometrics models, model’s parameters are introduced to the model either estimated or calibrated. After that, with regard to the parameters and information provided by the equations, the designed model simulates the variables and presents some of the features of the simulated variables (e.g., mean and standard deviation). A comparison of properties of real-world filtered variables and simulated variables can be a criterion for the success of the designed model. As such, given that the model is transformed into a linear logarithm, we evaluate the conventional methods in the DSGE literature. Stable values of variables converted to stable state are equal to zero; therefore, the standard deviation of some variables are compared (comparison of real and simulated variables). Afterwards, the reaction functions are evaluated in reaction to exogenous shocks.

2 Literature Review

Stock returns are affected by different variables and their anticipation is complicated. In fact, one of the important challenges of researchers and scientists in the field of “decision-making” and predicting is selecting variables that affect the decision-making and anticipation output. Various models have been used to assess factors affecting stock returns, including the DSGE, which is mostly applied to evaluate “the effect of monetary shocks on stock market index”. The DSGE model is introduced based on the methodology of Kydland and Prescott [9], where the behavior of different economic brokers are optimized considering target functions and their constraints [9]. This is one of the applicable models for analysis of fluctuations of financial and monetary economy changes, in a way that traditional models predicting macro econometrics are vulnerable to “Lucas Criticism”, which claims that the impacts of economic policies cannot be predicted with the use of historical data of a specific time period, where the policies were not used. Therefore, the DSGE models benefit from a natural criterion to evaluate the effects of policy change on welfare. With this background in mind, the present study aims to present a mathematical model for optimal portfolio based on the DSGE model.

Salahmanesh [15]: in a study entitled the design and calibration of a DSGE model, the scholars evaluate the dynamism of the stock market in Iran’s economy, concluding that financial markets, especially the capital market, can have strong links to other economic sections. With financial crisis and widespread downturn in the global economy, economists have again turned their attention to the functions of the financial markets. According to the studies performed in this field, a negative standard deviation of the price shock through the financial accelerator channel and bank capital leads to reduced production, consumption, investment, deposits and inflation. Therefore, macroeconomic variables are strongly
linked to stock market dynamics. Kaviani [7]: in a study, it is concluded that the basic monetary shocks have a primary positive effect on corporate stock price returns and will return to its stable state in the next rounds due to the decrease of shock. Moreover, due to the larger market share of companies in the capital market, the investment shock reduces stock price returns since there are more stocks on the market. However, in subsequent periods, stock prices will rise due to the increase in the expected profitability of these investments and the subsequent increase in price returns. Eslami Bidgoli and Bigdeloo [4]: the most important alternative investment opportunities in Iran are recognized and evaluated in this research. In fact, four investment opportunities of stock, land and housing, foreign exchanges, automotive, and gold and coin are introduced and factors affecting each of these variables are assessed. Following that, inflation is entered into the model to actualize and compare investment opportunities. An index is introduced for each investment opportunity to be able to compare the opportunities. Afterward, the comparison is carried out with two risk and returns criteria.

In addition, standard deviation and coefficient of changes of market is exploited to assess risks. Ultimately, these opportunities are considered alternatives to each other. Daliri and Mehregan [3]: in an article, the researchers investigate the effects of removing forced interest and conclude that: DSGE models are based on optimization and are rooted in the basics of microeconomics. For the first time, these models are used in 1980 to improve the function of the real business cycle (RBC). One of the problems of RBS is the competition of markets due to lack of nominal adhesion, where only real factors (e.g., productivity shock) can affect the fluctuations in the real section. This topic does not match the empirical belief of monetary economists. Therefore, New Keynesian economics forms the basis for the creation of DSGE models by entering the basics of microeconomics, adhesions, and incomplete competition into the models. The most important advantage of these models is their closeness to the real economic structure owing to their random and dynamic nature and their ability to determine the uncertainties and dynamics of economics in the form of mathematical models. Specialized studies on the structure of the banking industry in the Iranian economy suggest that the industry is "monopolistic competition". Furthermore, the function of monetary policy reaction in the Iranian economy will be a kind of "policy-making rule". Since Iran's monetary policy-maker behavior is "discretionary," one cannot expect it to reflect well on Iran's economic realities [17].

Importance of investment: investment is defined as postponing current consumption to achieve more consumption in the future. Increasing production and increasing economic growth will not be possible without increasing capital. In economic theories, the causes of underdevelopment of some countries include a shortage of capital and investment and the "vicious circle" of the lack of production is attributable to a lack of investment. Increased investment can help maintain purchasing power against inflation. Therefore, investment in both aspects of "supply and demand of capital" is essential for growth and development. The investment process has attracted inefficient capital (wandering liquidity) and directed it to the productive sectors of the economy. In addition, decision-making of policy-makers based on "risk and returns" and selecting the optimal portfolio provide optimal resource allocation. Top monetary officials warn of "long-term profitability of unproductive opportunities". Variety of alternative investment opportunities: investment is defined as postponing current consumption to achieve more consumption in the future. In investment, it is more important to identify opportunities through which more funds can be used for future consumption. In developing countries, a significant portion of savings are kept in cash or in the form of "coins and gold", "foreign exchange currency", "housing", "land", "automobile", "bank deposit", and "stock purchase", the main cause of which is the fear of increased prices, lower purchasing power and predicting a decline in the value of individuals' assets in the future.

How to select optimal investment opportunities for investors: opportunities that bring more profitability
to the investor are considered to be a better investment. "Profitability of investments" is more important than various factors that may direct the tendency of investors toward a particular economic sector. The incentive to gain profits lays the foundation for attracting capital in such sectors. Investors implicitly or explicitly invest at their own risk, and those accepting high risks expect high returns as well. On the other hand, investors accepting smaller risks expect lower returns. In general, investors seek to maximize their “returns” through creating a reasonable relationship between risk and their investment returns.

**Importance of the topic for Iran:** Scientific research shows that while there are considerable liquidity and savings in Iran, they do not move towards productive investment and corporate finance. According to objective evidence, in developing countries (e.g., Iran), a significant portion of savings are held in cash or the form of gold, coins, jewelry, and foreign exchange currency mainly due to the fear of the future and predicting a decline in the value of those assets. Iran is one of the countries facing high liquidity growth rates, not used in the productive sectors of agriculture and construction and directed toward alternative markets such as land, housing, automobile, currency, and gold due to instability, sanctions, war, crisis, and natural disasters to maintain the value of their assets.

Since the Central Bank and the Monetary and Credit Council, which are among the highest monetary authorities in Iran, are not independent and do not play an important role in the money market, the money market has little influence on important monetary variables such as currency maintenance, inflation control, and interest rate stability. Therefore, paying attention to the financial markets and the capital market and investing in stocks and securities is of great importance. “Presenting a mathematical model that can introduce the optimal portfolio to investors” in ways that investors (e.g., families and individuals who save their money) become motivated to invest in the capital market, and there would be an increase in the volume of capital market in Iran.

In this respect, development of the stock market and diversity in providing securities and economic agencies could financially support economic firms through the capital market, which increases the importance of investment in the economy of Iran more than before. Investors aim to gain “the possible maximum return” from their investment. Therefore, they constantly seek information that would help them decide about the optimal allocation of funds and select a suitable investment market. “Economic-financial information” have high effectiveness and importance. For instance, inflation is high in Iran, and investors want to know if stock investments can hedge against inflation and maintain the value of their assets or not. In addition, the analysis of financial statements and the "qualitative features of information" provided in the financial statements are of paramount importance and greatly increase the chance of successfully investing in the stock market. In fact, familiarity with financial statement analysis techniques, financial statement items, tools, techniques, liquidity ratios, activity and profitability ratios, and interim financial statements can increase the knowledge of investors regarding selecting an optimal portfolio and achieving maximum returns [4].

**Research background:** Taylor [19] evaluates the dynamics of the stock market as a reflection of economic uncertainty and the valuation of financial wealth. In the mentioned study, the researcher expresses the effect of wealth on the mechanisms of the actual economic sector. Bernanke and Gertler [1] introduce the relationship between macroeconomics and dynamism of the stock market to be relying on inflation, confirms the role of capital market in macroeconomics through the effect of wealth on total consumption and demand and evaluates the monetary policy-makers’ behaviors in dealing with big jumps in stock prices. Moreover, Nestico performs financial modeling of the household sector for optimization. Applying dynamic DSGE models and VAR . show that forecasting models are able to transform long-term financial returns. Gupta and Modise [26]. use the DSGE model, concluding that while inflation rates have very strong predictive power over short
six-month time horizons, interest rates have different significance in explaining stock return behavior, demonstrate that stock price fluctuations are important variables for financial demand alongside the standard variables and stock prices. In addition, investor risk avoidance is an important force for investors to move to safe assets during stock price volatility. Chung & Ariff [25] conclude that changes in the money supply lead to a positive liquidity effect and liquidity changes have a positive effect on stock prices. In the end, they confirm the theory that money supply affects liquidity, and liquidity affects the price of non-bank stocks. Chung and Chow [24] confirm a negative relationship between market fluctuations and stock returns. In general, there are different models to evaluate the factors affecting stock returns and investment, including the DSGE model, which is mostly used to assess the impact of economic shocks on stock market index.

The present study aims to evaluate the effect of productivity shock and foreign exchange earnings shock on stock returns of companies with the DSGE model. The DSGE model is presented based on the methodology of Kydland and Prescott [9]. Since the DSGE models optimize the behavior of different economic agents with respect to the objective functions of each, they are among applicable models for analyzing fluctuations in financial and monetary economics changes. Given the vulnerability of traditional models of macroeconomic prediction against Lucas Criticism (which claims that the effects of economic policies cannot be predicted using historical data from a period when that policy (game rules) did not exist), DSGE models benefit from a natural criterion to assess the effects of policy change on welfare Tower [23]. Notably, the current study’s main question is: what is the effect of productivity shock and foreign exchange earnings shock on stock returns and optimal portfolio within the framework of the DSGE model.

3 Model Specification: Extraction of Equations of DSGE Model

Household: It is assumed that the economy is made up of a large number of households, which is represented by index (i) in the present study, and all of them are homogenous. Households gain utility by consuming goods and maintaining the real cash balance and excessive presentation of goods reduces utility since it leads to reduced leisure. The value of utilities of the representative household during its life is presented, as follows:

$$E_i \sum_{t=0}^{\infty} B^t u_i(t)$$

Where (β) is the time actualization factor. In addition, the function of household utility, which is a function of total household consumption, real cash balance, and work supply, is presented below:

$$u_i^t = \left[ \frac{1}{1 - \sigma_c} (c^t - h_{c,t-1})^{1-\sigma_c} - \frac{1}{1 + \sigma_i} (l^t)^{1-\sigma_i} + \frac{1}{1 - \sigma_m} \left( \frac{M_{ct}}{p^t_c} \right)^{1-\sigma_m} \right]$$

In Equation 2, consumer goods are a combination of different domestically produced and imported consumer goods, established by national producers and import. In utility function 2, δc expresses the relative avoidance risk factor that shows the inverse of the intertemporal elasticity of substitution, δ_i is indicative of the inverse of the human resource supply elasticity, and δ_m is the inverse of the real cash balance elasticity (m_{ct} = \frac{c_{ct}}{p^t_c}), compared to interest rate. The utility function in Equation 2 reflects the external habits (keep up with the joneses) of consumers that depend on average per capita consumption...
of the economy. Therefore, when the consumption of each representing family in the economy at (t) time is higher than the (h) percentage of average per capita economy in the (t-1) period, higher consumption leads to positive utility, where (h) demonstrates the extent to which the consumer tends to level its consumption relative to the average per capita consumption of the previous period. The higher the (h), the more it shows a high degree of dependence on consumption habits.

Selecting a Consumer Basket and Obtaining Consumer Demand Functions: In Equation 2, it is hypothesized that the total consumption at real price ($c^d_t$) is a combination of consumption of domestic goods ($c^d_t$) and imported goods ($c^m_t$), provided by domestic and importing manufacturing firms, respectively. The goods are combined by a Dixit–Stiglitz model, meaning:

$$c_t = \left[ \frac{1}{\varepsilon_c} \left( \frac{c^d_t}{p^d_t} \right)^{\mu_{c-1}} + \left( 1 - \varepsilon_c \right) \frac{1}{\mu_c} \left( \frac{c^m_t}{p^m_t} \right)^{\mu_{c-1}} \right]^{\mu_c}$$

(3)

Where ($\varepsilon_c$) and ($1-\varepsilon_c$) are the share of domestic and imported goods in the total household consumption basket, respectively, and ($\mu_c$) is the elasticity of substitution between consumer and imported goods. In general, the issue of household decision-making can be evaluated at two stages. In the first stage: the household decides about the combination of the consumer products to minimize the cost of obtaining a certain level of consumption of combined goods. At this stage, families minimize the cost of purchasing a combined use ($c_t$). In the second stage: with regard to the cost of access at any given level of consumption ($c_t$), the household selects an optimal amount of ($\frac{m^c_t}{p_t}$) and ($L_t$) and ($c_t$), in a way that utility is maximized. In order to perform the first stage, families minimize the cost of purchasing combined consumption level ($c_t$). In addition, they solve the following equation regarding selecting domestic and imported consumer goods:

$$\text{Min} \quad c_t^d p^d_t c^d_t + p^m_t c^m_t$$

s.T: $c_t = \left[ \frac{1}{\varepsilon_c} \left( \frac{c^d_t}{p^d_t} \right)^{\mu_{c-1}} + \left( 1 - \varepsilon_c \right) \frac{1}{\mu_c} \left( \frac{c^m_t}{p^m_t} \right)^{\mu_{c-1}} \right]^{\mu_c}$

(4)

Equation 4: where ($c^d_t$) and ($c^m_t$) are the consumption of domestic and imported products, respectively, and ($p^d_t$) and ($p^m_t$) are the prize index of domestic and imported products, respectively. The Equation 4 and extraction of F.O.C of the first-rank conditions can be used to retrieve demand functions for domestic and government consumer goods, as follows:

$$c^m_t = \left( 1 - \varepsilon_c \right) \left( \frac{p^m_t}{p^d_t} \right)^{-\mu_c} c_t$$

(5)

$$c^d_t = \varepsilon_c \left( \frac{p^d_t}{p^m_t} \right)^{\mu_c} c_t$$

(6)

The total consumer price index ($p^c_t$) and its components are obtained by placing equations 5 and 6 in the household consumer basket ($p^d_t c^d_t p^m_t c^m_t = p^c_t c_t$). Total consumer price index ($p^c_t$):

$$p^c_t = \left[ \varepsilon_c \left( \frac{p^d_t}{p^m_t} \right)^{1-n_c} + \left( 1 - \varepsilon_c \right) \left( p^m_t \right)^{1-n_c} \right]^{\frac{1}{1-n_c}}$$

(7)

Where ($p^c_t$) indicates the total consumer price index. After determining the optimal combination of goods in the first stage, the goal of households in the third stage is to maximize their expected utility function compared to interim budget constraints. In the second stage and after determining the optimal combination of goods in the first stage, the goal of households is selecting optimal amounts of con-
sumption ($c_t$), workforce ($L_t$) and financial assets, in a way that utility is maximized. Household financial assets consist of "money", "equity securities", and "stocks". While money is entitled to no benefits, equity securities are entitled profit (interest) at ($r_{1t}^d$) rates. In addition, stocks are allocated dividends (if there is any) and capital income. The household financial assets at the end of the ($t$) period include cash, equity securities, and stocks ($N_t(j)$) published by the jth intermediary firm. The nominal price of each share of the jth firm in the ($t$) period is shown with ($p_t^x(j)$). Therefore, wealth of the ith household shares includes a basket of stocks of intermediary agencies, where each stock has a dividend with a nominal value of ($D_t j$). As such, at the beginning of each period, households' sources of income include financial wealth from net wage rent, capital, and a set of financial wealth from the previous period (e.g., money, equity securities, and stocks). We use the Nestico studies [27] to model the stock assets. The assets (wealth) of the jth household shares existing from the previous round ($\Omega_{t-1}^J$) can be presented, as follows:

**Stock Asset Modeling Based on “Nestico Study”:**

$$\Omega_{t-1}^J = \int_0^1 [p_t^x(j) + D_t^x(j)] N_t(j) d_j$$  \hspace{1cm} (8)

Since the beta coefficient (systematic risk) is defined by the sensitivity of stock returns (stock price changes) to market returns (price return or equity price index), the following equation is established: B coefficient estimation

$$B_t^l = \frac{\Delta (p_t^x - p_{t-1}^x)}{\Delta (p_m - p_{m-1})}$$  \hspace{1cm} (9)

The interim periodic budget of households in terms of real prices can also be expressed as follows:

**Interim Periodic Budget of Households**

$$\frac{1}{\pi_t^x} p_t^x (\int \frac{w_t^l}{c_t^l} d_t + m_t^c) = (1 + r_{t-1}^d) \frac{b_t^l}{\pi_t^x} + \frac{m_{t-1}^c}{\pi_t^x} + \frac{1}{\pi_t^x} \Omega_t^{l,j} + T_t^l - T_{t-1}^l + y_t^l c_t^l + l_t^l + b_t^l$$  \hspace{1cm} (10)

Where ($l_t^l$) is investment level, ($b_t^l$) is equity securities, ($r_{t-1}^d$) shows the nominal interest rate of equity securities, ($T_t^l$) is a household tax (direct and indirect taxes and value-added tax), ($T_{t-1}^l$) is government subsidy payments, ($p_t^l$) is investment cost, and the household keeps its wealth in the form of real money balance ($M_t^{c,l}$) and equity securities ($h_t^l$). In addition, ($\pi_t^x$) is the inflation rate based on total consumer price index and ($\pi_t^x$) is the stock price shock, which actually constitutes the price bubble. Other variables are previously defined in the text, and ($y_t^l$) shows the income of households, as defined Below:

$$y_t^l = \frac{w_t^l}{p_t^l} l_t^l + R_t^k z_t^l k_t^l - \psi (z_t^l) k_t^l + Div_t^l$$  \hspace{1cm} (11)

The total income of households encompasses labor wages ($\frac{w_t^l}{p_t^l} l_t^l$), lease of capital minus the cost related to changes in capital utilization rate, and dividends of firms producing intermediate goods ($Div_t^l$). In Equation 11, ($w_t^l$) is nominal wages, ($R_t^k$) is the rate of real return on capital, and ($z_t^l$) is the intensity of using (operation rate) the capital capacity, and ($\psi (z_t^l)$) is the cost of operating capital. The cost of exploiting the capital capacity ($\psi (z_t^l)$) is indicative of the cost of each unit of physical capital. The equations are established in long-term balance mode. Capital Stock and Investment: the capital stock is in the households’ ownership and is used as homogeneous production factor in the production process. Households rent their capital stock with the rate of ($R_t^k$) to agents producing intermediate goods. Capital can be increased in two ways:

1) Through increasing investment ($I_t$), which increases capital stock.
2) Changing the level of exploitation of the capital stock.

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It is hypothesized that the capital accumulation process is carried out by the following equation:

**Capital Accumulation:**

\[ k_t = (1 - \delta)k_{t-1} + \left[ 1 - s \left( \frac{L_t^t}{L_{t-1}^t} \right) \right] l_t^e_k \]  

(12)

**The behavior of finished goods manufacturers:** an example firm similar to what is assumed in Ireland [28], manufactures finished goods, where \( Y_{jt} \) units purchase \( J \in [0,1] \) intermediary goods with the nominal prize of \( P_{jt} \) and produce the finished product \( Y_t \). According to the equation below, which is a collector based on the Dixit–Stiglitz model, we can write:

**Dixit–Stiglitz model:**

\[
\int_0^1 Y_{jt} \left( \frac{\theta - 1}{\theta} \right) d_j \left( \frac{1}{\theta - 1} \right) \geq Y_t
\]

(13)

Where \( (\theta > 1) \) and intermediary goods distinct and incomplete replacements of each other and there is a fixed elasticity of substitution \( (\theta) \) among them. Therefore, in the \((t=0,1,2,3000)\) period, the example finished goods manufacturing firm selects \( y_{jt} \) for all \( j \in (0,1) \) in a way that it can maximize its profit.

\[ Y_{gt} = \left[ \frac{P_{ht}}{P_t} \right]^{-\theta} Y_t \]  

(14)

Where \( (-\theta) \) shows price elasticity of demand for \( (j) \) intermediary goods. In competitive markets, the finished product manufacturing firm has zero profits. The condition of zero profit \( (P_t) \) is defined, as follows:

\[ t=0,1,3, \ldots \text{ for all } \]  

\[ p_t = \left[ \int_0^1 P_{jt}^{1-\theta} d_j \left( \frac{1}{3-\theta} \right) \right] \]  

(15)

**The behavior of intermediary goods manufacturers:** the chain economy consists of monopolistic competition firms in the intermediate goods sector that are indexed in the range \((0 & 1)\). Each firm produces distinctive goods. In fact, these firms produce intermediary goods \( (j) \) by using the workforce, capital, and other units. These firms use workforce and capital institutions as entities in the process of production. Given the presence of important development budgets for private sector productivity due to the government’s dominance over the economy, it is, therefore, necessary to consider the formation of state capital in the form of a function of the intermediary goods manufacturing firms. The production function of intermediary goods manufacturing firms in the form of Cobb-Douglas function is presented below. **Cobb-Douglas Production Function:**

\[ y_t^j = A_t (z_t^j k_{t-1}^j)^{\alpha} (L_{t-1}^j)^{1-\alpha} (k_{t-1}^G)^k \]  

(16)

Effective capital stock

\[ z_t^j k_{t-1}^j = \bar{k}_{t-1} \]

State capital

\[ k_{t-1}^G \]

Effective capital stock and formation of the state capital are as shown above, and are assumed common to all firms in this sector. In addition, \( (A_t) \) is indicative of productivity, which is common for all firms and is assumed to follow the process presented below:

\[ A_t \] shows productivity that is common for all firms.

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1 Equations of first order conditions, household savings and consumption behavior, demand for money, capital accumulation and investment, and dynamics of firm stock prices are not expressed for briefness, and authors are able to send them to readers as needed.
The jth intermediary goods manufacturer seeks to minimize its costs according to a certain amount of production. Therefore, the objective function of the jth firm is, as follows. The objective function of the jth firm:

\[
\text{Min } k_{t-1}.L_{t} \frac{w_{t}}{p_{t}^{i}} L_{t}^{j} + R_{t}^{k} z_{t} k_{t-1}^{j} \\
\text{s.t: } y_{t}^{j} = A_{t}(z_{t} k_{t-1}^{j})^{a} (L_{t-1})^{1-\alpha} (k_{t-1})^{b}
\]

Where \((w_{t})\) is nominal wage, \((P_{t}^{j})\) is the return of investment, and \((y_{t}^{j})\) is demand for the jth goods. If we obtain the first-order terms for the firm’s optimization problem, then the firm's cost in terms of real prices can be written as follows. Marginal Cost of Firm:

\[
M_{t}^{i} = \frac{\text{Mc}_{t}}{p_{t}} = \frac{1}{A_{t}} \left[ \frac{1}{1-\alpha} \right]^{1-\alpha} \left[ \frac{w_{t}}{p_{t}} \right]^{1-\alpha} (R_{t}^{k})^{a} (k_{t-1})
\]

Another issue facing the intermediary firm is a price adjustment. In this study, we apply the calvo method [29], to adjust the prices. It means that in each period, \((1 - \theta_{p})\) of firms are able to optimally adjust their prices. Other firms \((\theta_{p} \%)\) that cannot optimally adjust their prices in the current period, partially index the prices using the equation below. Price Adjustment:

\[
p_{t+1}^{i} = \left( \pi_{t}^{d} \right)^{T_{p}} p_{t}^{i}
\]

Where \(\pi_{t}^{d} = \frac{p_{t}}{p_{t-1}}\) is indicative of the inflation rate of products. The \(( T_{p} i )\) is a parameter that indicates the degree of price indexing. The price determined by the ith firm at t time is a function of the expected final costs of the future and is equal to a markup on balanced final costs. In the case of completely flexible costs \((\theta_{p} = 0)\), the markup at t time is equal to \((\frac{\xi}{\xi-1})\), where \((\bar{p} = \xi) \text{Mc}^{i}_{t}\) is established, which is the condition of monopolistic competition in a state of complete price flexibility, where the price is equal to markup plus the final nominal price. However, in the case of nominal rigidity \((\theta_{p} > 0)\), the markup value changes over time when the economy faces endogenous momentum. Since only \((1 - \theta_{p})\) percent of firms can optimally adjust their prices at each period, and other firms index prices based on the price of the previous periods, the total price index at t time is based on the following weighted-average formula using the following equation. Total Cost Index at T Time:

\[
[p_{t}^{i}]^{1-\xi} = \theta_{p} \left[ \left( \pi_{t-1}^{d} \right)^{T_{p}} p_{t-1}^{i} \right]^{1-\xi} + (1 - \theta_{p})[\bar{p}]^{1-\xi}
\]

Government and central bank: government: similar to the research by Calvo [29], for low-income developing countries with oil income, and the study by Calvo [29]. for Ghana, the real budget constraint of the state budget is determined by the following equation. Government Budget Constraint to Real Price:

\[
g_{t} + \frac{(1 + r_{t-1})b_{t-1}}{\pi_{t}^{d}} = \frac{w.EX_{t}.\alpha_{t}}{p_{t}^{d}} + T_{t} + other_{t} + fa_{t} + \frac{GBD_{t}}{p_{t}^{d}}
\]

Where \((g_{t})\) is the total government expenditure, \((EX_{t})\) is the nominal exchange rate, \((0_{t})\) is foreign exchange earnings, \((b_{t})\) is equity securities, \((T_{t})\) is tax revenues, \((other)\) is other incomes, \((fa_{t})\) is the assignment of state-owned companies, and \((GBD_{t})\) is the state budget deficit. As is clear, the government spends \((w)\) percent of oil revenue through the budget. Monetary policy: it is assumed that monetary policy is in a way that according to which, the policy-maker determines the debt growth rate of...
banks to the central bank in a “completely discretionary” form to reach two goals of: 1) reduction of production deviation from potential production, and 2) inflation deviation from target inflation. In addition, it is assumed that the central bank has no explicit inflation targeting to be made public. However, due to targeting in development programs, policymakers always attempt to pursue an implicit goal. Monetary policy response function (in the linear-logarithm form) will be as follows: **Monetary policy response function (in the linear-logarithm form):**

\[
\hat{\theta}_t = \rho \hat{\theta}_{t-1} + \theta_\pi \hat{\pi}_t + \theta_\gamma \hat{\gamma}_t + \theta_{rer} \hat{rer}_t + \theta_{\pi^2} \hat{\pi}^2_t + \epsilon^\theta_t
\]

(23)

\[
\hat{\pi}_t = \bar{M}_t^ε - \bar{M}_{t-1}^ε + \hat{\pi}_t^ε
\]

(24)

\[
\epsilon^\theta_t = \rho \epsilon^\theta_{t-1} + u^\theta_t \sim N(0, \delta^\theta_0)
\]

(25)

Where \(\hat{\theta}_t\) is the monetary base nominal growth rate, \((\hat{rer}_t, \hat{\gamma}_t, \hat{\pi}_t)\) are deviation of inflation rate, and production logarithm, and real exchange rate from their steady state values, respectively, \(\theta_\pi, \theta_{rer}, \theta_\gamma, \theta_{\pi^2}\) are the coefficient of importance that policy makers take into account for inflation, production, exchange rate, and total stock price index, respectively, and \(\epsilon^\theta_t\) is the monetary policy momentum that follows a random process (AR(1)). **Market Balance:** The final good market is in equilibrium when production equals household demand for consumption and investment, government spending and export minus imports:

\[
y_t = c_t + i_t + y_t + \frac{ex_t(p_t^c x_t + \alpha_t)}{p_t^c} - \frac{p_t^{mc} c_{t nm} + p_t^{mc} y_{t nm}}{p_t^c}
\]

(26)

The amount of total production is equal to non-oil and oil production as follows:

\[
\text{Total Production Amount } Y_t = \left[\frac{1}{\alpha^\mu} (Y^\mu_0)^{\frac{\mu_0}{\mu_1}} (1 - \alpha^\mu) \frac{1}{\mu^\alpha} (Y^0_\alpha)^{\frac{\mu_0-1}{\mu_0}} \right]^{\frac{\mu_0}{\mu_0-1}}
\]

(27)

### 4 Statistical Data and Calibration of Parameters

The two shocks presented in this study illustrate the impact of the productivity shock and the exchange rate (foreign exchange earnings). The production gap is defined as the deviation of the logarithm of actual production from potential production. In addition, the potential production is calculated using the Hodrick-Prescott (HP) filter. According to the definition of growth rate in the New Keynesian economics, the variable growth rate is defined as the variable ratio in (t) period to variable in (t-1) period. Since all variables in the model are defined as the logarithmic deviation of the steady state value, inflation rate, and money base growth rate are obtained from the HP filter with \(\lambda=677\) logarithm of the ratio of variable to the amount of the previous period. Before estimating parameters, those that do not need estimation must be specified and their values must be calibrated. However, estimation of some of the parameters is not required due to being extracted from the steady-state values of the variables. Some other parameters are also at the steady-state of the model, compared to the variables. The parameters able to be calibrated based on economic data of Iran are summarized in Table 1.

### 5 Model Validation

In order to evaluate the success level of the model, we compare the estimated torques, parameters, and ratios with their corresponding values in the real world. Afterward, by drawing the instantaneous reaction and MCMC graphs, it was clear that the closer the ratios, the higher the model’s convergence and success.
<table>
<thead>
<tr>
<th>Symbol</th>
<th>Parameters</th>
<th>Value</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha$</td>
<td>Coefficient of capital on production</td>
<td>0.42</td>
<td>Shahmoradi [30]</td>
</tr>
<tr>
<td>$\sigma_1$</td>
<td>Inverse elasticity of workforce to real wages</td>
<td>2.92</td>
<td>Taei, [31]</td>
</tr>
<tr>
<td>$\delta_c$</td>
<td>Inverse elasticity of intertemporal consumption substitution</td>
<td>0.8</td>
<td>Kavand, [32]</td>
</tr>
<tr>
<td>$\delta_m$</td>
<td>Inverse elasticity of real money balances</td>
<td>1.315</td>
<td>Zanganeh,[33]</td>
</tr>
<tr>
<td>$\varphi$</td>
<td>Elasticity of investment adjustment cost function</td>
<td>3.943</td>
<td>Rahbar et al. [34]</td>
</tr>
<tr>
<td>$h$</td>
<td>Stability degree of habits</td>
<td>0.3</td>
<td>[35]</td>
</tr>
<tr>
<td>$\beta$</td>
<td>Consumer time preference rates</td>
<td>0.95</td>
<td>Research findings</td>
</tr>
<tr>
<td>$\psi$</td>
<td>Inverse elasticity of cost function relative to exploitation cost</td>
<td>0.21</td>
<td>Rahbar et al. [34]</td>
</tr>
<tr>
<td>$\tau_p$</td>
<td>Price indexing degree</td>
<td>0.511</td>
<td>Rahbar et al. [34]</td>
</tr>
<tr>
<td>$\theta_p$</td>
<td>Percentage of firms unable to adjust their prices</td>
<td>0.20</td>
<td>Parsa et al. [21]</td>
</tr>
<tr>
<td>$\eta_c$</td>
<td>The elasticity of substitution between domestic consumer goods and imported goods</td>
<td>1.05</td>
<td>Parsa et al. [21]</td>
</tr>
<tr>
<td>$\mu^0$</td>
<td>The elasticity of substitution between oil and non-oil production</td>
<td>0.15</td>
<td>[36]</td>
</tr>
<tr>
<td>$\rho_i$</td>
<td>Autoregressive process coefficient of investment momentum</td>
<td>0.28</td>
<td>[36]</td>
</tr>
<tr>
<td>$\bar{w}_e$</td>
<td>Inflation significance coefficient in the monetary policy response function</td>
<td>-1.548</td>
<td>[38]</td>
</tr>
<tr>
<td>$\bar{w}_p$</td>
<td>Production significance coefficient in the monetary policy response function</td>
<td>-1.70</td>
<td>[38]</td>
</tr>
<tr>
<td>$\bar{w}_s$</td>
<td>Significance coefficient of total stock price index in the monetary policy response function</td>
<td>0.9</td>
<td>Bayat et al. [37]</td>
</tr>
<tr>
<td>$\bar{w}_{rer}$</td>
<td>Significance coefficient of real exchange rate in monetary policy response function</td>
<td>0.8</td>
<td>[39]</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Parameters</th>
<th>Value</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta_s$</td>
<td>Saver Household Discount Factor</td>
<td>0.974</td>
<td>Taghipour [43]</td>
</tr>
<tr>
<td>$\beta_b$</td>
<td>Borrower Household Discount Factor</td>
<td>0.947</td>
<td>Research findings</td>
</tr>
<tr>
<td>$R_d$</td>
<td>Gross real seasonal interest rate on deposits</td>
<td>1.037</td>
<td>Central bank</td>
</tr>
<tr>
<td>$R_h$</td>
<td>Gross real seasonal interest rate on loans to households</td>
<td>1.055</td>
<td>Central bank</td>
</tr>
<tr>
<td>$R_p$</td>
<td>Quarterly gross real rate of return on stock market</td>
<td>1.067</td>
<td>Research findings</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>Relative risk aversion coefficient of whole households</td>
<td>1.571</td>
<td>Tavakolian [18]</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>Capital share in the production function</td>
<td>0.42</td>
<td>Tavakolian [18]</td>
</tr>
<tr>
<td>$\delta_e$</td>
<td>Fixed capital depreciation rate</td>
<td>0.042</td>
<td>Amini &amp; Neshat Haji [39]</td>
</tr>
<tr>
<td>$\beta_e$</td>
<td>Entrepreneurs discount factor (wholesale)</td>
<td>0.959</td>
<td>Research findings</td>
</tr>
<tr>
<td>$R_e$</td>
<td>Nominal seasonal interest rates for wholesale loans</td>
<td>1.065</td>
<td>Central bank</td>
</tr>
<tr>
<td>$\theta_R$</td>
<td>Retail price stickiness index</td>
<td>0.5</td>
<td>Tavakolian [18]</td>
</tr>
<tr>
<td>$\gamma_p$</td>
<td>The degree of price indexing of finished goods</td>
<td>0.715</td>
<td>Tavakolian [18]</td>
</tr>
<tr>
<td>$\beta_B$</td>
<td>Banking discount factor</td>
<td>0.974</td>
<td>Research findings</td>
</tr>
<tr>
<td>$rr$</td>
<td>Legal reserve rates</td>
<td>0.125</td>
<td>Central bank</td>
</tr>
<tr>
<td>$k_L^D / D_t$</td>
<td>Banks' capital to deposit ratio</td>
<td>0.11</td>
<td>Dargahi, Hadian [40]</td>
</tr>
<tr>
<td>$p_e$</td>
<td>Money growth rate coefficient</td>
<td>0.82</td>
<td>Ahmadian [41]</td>
</tr>
<tr>
<td>$D / MB$</td>
<td>The ratio of deposit to monetary base</td>
<td>4.3</td>
<td>Dargahi, Hadian [40]</td>
</tr>
</tbody>
</table>
Table 3: Stable and Long Run Values of Model Variables

<table>
<thead>
<tr>
<th>Variable</th>
<th>Value</th>
<th>Variable</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$y$</td>
<td>1.9</td>
<td>$d$</td>
<td>1</td>
</tr>
<tr>
<td>$\pi$</td>
<td>0.45</td>
<td>$\pi$</td>
<td>0.9</td>
</tr>
<tr>
<td>$k_e$</td>
<td>6.7</td>
<td>$\bar{q}$</td>
<td>2.4</td>
</tr>
</tbody>
</table>

Table 4: Comparison of Real and Simulated Calculated Torques

<table>
<thead>
<tr>
<th>Variable</th>
<th>Mean Real</th>
<th>Mean Simulated</th>
<th>Standard Deviation Real</th>
<th>Standard Deviation Simulated</th>
<th>First-order Autocorrelation Real</th>
<th>First-order Autocorrelation Simulated</th>
</tr>
</thead>
<tbody>
<tr>
<td>GDP</td>
<td>0.091</td>
<td>0.076</td>
<td>0.062</td>
<td>0.0518</td>
<td>0.801</td>
<td>0.8790</td>
</tr>
<tr>
<td>Private consumption</td>
<td>0.068</td>
<td>0.08</td>
<td>0.08</td>
<td>0.715</td>
<td>0.875</td>
<td>0.8802</td>
</tr>
<tr>
<td>Capital stock</td>
<td>0.006</td>
<td>0.003</td>
<td>0.0088</td>
<td>0.0091</td>
<td>0.96</td>
<td>0.9974</td>
</tr>
<tr>
<td>Deposit</td>
<td>0.072</td>
<td>0.006</td>
<td>0.066</td>
<td>0.0792</td>
<td>0.537</td>
<td>0.6145</td>
</tr>
<tr>
<td>Inflation</td>
<td>0.005</td>
<td>0.001</td>
<td>0.04</td>
<td>0.0388</td>
<td>0.461</td>
<td>0.4305</td>
</tr>
<tr>
<td>Stock price</td>
<td>0.078</td>
<td>0.034</td>
<td>0.246</td>
<td>0.3220</td>
<td>0.674</td>
<td>0.6929</td>
</tr>
</tbody>
</table>

6 Simulation Results

using the parameters and calculating some parameters with the application of economic data of Iran, the linear-logarithm equations system is simulated in MATLAB applying Dynare Software, the basis of which is evaluated according to the analysis of effects of “productivity” and “foreign exchange earnings fluctuation” shocks on variables of macroeconomics and assessment of “risk and returns” of alternative investment opportunities. In addition, model simulation results are evaluated using the parameters estimated by “Bayesian method” and by calculating some parameters applying the economic data of Iran. These simulation results are presented below:

6.1 Effects of Productivity Shock (Technology)

the family usually maximizes its utility of consuming goods and services and combining work and leisure and, ultimately, savings as an investment in the next period due to budget constraints. Labor productivity shock and final labor income can change this utility maximization. The fluctuation in the final labor income can disrupt the optimization or maximization of household utility. Productivity shock results can be visualized through instantaneous reaction graphs by drawing Markov Chain Monte Carlo (MCMC) graphs, which in fact represent the reaction of real variables to productivity shocks.

Fig. 1: Instantaneous Reaction Functions of Financial and Economic Variables to Monetary Base Shock
(Source: Researcher’s Findings)
7 Conclusion and Recommendations

Financial markets, especially the capital market, are a logical and systematic market with strong links to other sectors of the economy. With the financial crisis in 2007 and 2008 and the widespread downturn in the world economy, economists are again focusing on how financial markets work. The present study aims to design and calibrate a new Keynesian DSGE model by investing in stocks and securities markets and other alternative markets considering the risk and returns. In the end, we evaluate the effect of shocks on macroeconomics variables. In the designed model, households (consume, provide labor, invest with their savings, and hold a portfolio of stocks, cash, securities and other assets based on risk, return or optimal portfolio). In short, households save money in cash, bank deposits, stocks and securities, and eventually in their investments in other markets such as buying land, gold and coins, foreign currency, and cars.

References


[37] Bayat, M., Reaction of pricing statistics thresholds to inflation rate changes, Monetary and Banking1990, Research Institute of the Central Bank of the Islamic Republic of Iran.

