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A Quantum Model for the Stock Market

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ABSTRACT

Price return and P/E are two interesting factors for a lot of investors; The Bohmian quantum mechanics used referring to the time correlation of return and P/E of the stock market under consideration. In this study, we extend the quantum potential concept to determine the behaviour of P/E and also price return in two different industry of Tehran stock market during a time interval of April. 2008 to march 2019. The obtained results show that the quantum potential behaves in the same manner for P/E and price return, also confines the variations of the P/E and price return into a specific domain. Furthermore, a joint quantum potential as a function of return and P/E is derived by the probability distribution function (PDF) constructed by the real data of a given market. It serves as a suitable instrument to investigate the relationship between these variables. The resultant PDF and the corresponding joint quantum potential illustrate that where we have light points in joint quantum potential chart, the probability of those amount of P/E and price return are more than other points. In addition, because of the rectangular shape of the joint quantum potential chart we can say that these two variables behave as two independent variables in the Market.

1 Introduction

Predicting the trend and the variation of P/E ratio and stock return has attracted the attention of many finance and economics researchers. Nowadays evidence shows that some stock market studies have focused on stock price and its behaviour over the time. However, due to the variability and non-stationary trend of stock price, most researchers mainly focus on stock return, defined as the logarithm of the relative change of price [1,2], rather than stock price or raw price return. When the P/E of a stock market is concerned, different definitions are proposed by the relevant literature, including the earning of per share in each day and daily price return as well and the turnover ratio [3], etc. There are several reasons

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for the importance of discussing the relationship between P/E ratio and stock return as follows. First, the existing models in the financial markets can predict the P/E and price return trend in the market . Thus, exploring the amount of P/E ratio and stock return may help distinguish and decide between different hypotheses proposed about the market structure. Second, for those studies that use a combination of P/E ratio and stock return data, it is important to know how these two are interrelated. Third, the mechanism and modality of determining P/E ratio and return have important implications for future market studies; where the price changes have a considerable impact on the P/E ratio of futures contracts or not. The correlation between P/E ratio and price return and predicting these variables by the bohemian quantum potential model has been never studied and discussed by many authors, but there are different studies using the above equation for analyzing other variables in stock markets.

2 literature Review

There are some studies that are related to this study subject and using mathematics or physics in finance to search about different variables in stock markets. For example, Tripathi presented that mathematical relations are applied in financial management [4]. Conducting an empirical study and using the data from the New York Stock Exchange was found that daily price changes has no relation with trading volume for both in absolute terms [5]. Using OLS and GARCH models was concluded that there exists no Casual relation between volume and return for the Egyptian market [6]. Mantegna and Stanley gained new ideas about financial markets behaviour by implementing statistical and physical methods [7]. The chaos theory was applied to study the dynamics of a financial system by Chakraborti and his associates [8] and Chen [9]. Moreover, Baaquie and his colleagues studied the basic concepts of economics based on statistical mechanics, using classical potential and Hamiltonian dynamics [10,11].

During the last years, the correlation between the stock markets and their corresponding variables has increased to be inevitably tangled. This behaviour of the financial assets has pushed researchers to use the quantum potential model taken from Bohmian quantum mechanics. Employed the principles of quantum mechanics to describe the stochastic processes inherent in the financial markets. Similarly, Choustova employed Bohmian quantum mechanics as a theoretical framework to implement a model for describing the dynamics of financial systems [12].

Tahmasebi et al. did an empirical study that implemented Bohmian quantum mechanics in financial markets that presented the entanglement between today's and yesterday's prices of stock markets implies the existence of quantum potential which limits the price return changes into a specific domain [13]. Shen and Haven followed the same method by employing both classical and quantum potentials and finalized that, in addition to stock markets, there create potential walls for commodity markets as well [14]. the same technique to study the collective behavior of some targeted emerging and developed markets was used by Nasiri and his colleagues [15]. Using the empirical data of the market indices, they expressed that the quantum potential walls limit the variations of the price return into a definite interval where the distance between the walls can be a representative for the risk of the respective stock index.

In this study, we follow the logic adopted by Tahmasebi et al. [13] and Shen and Haven [14] and use the data extracted from top 10 Tehran stock market companies, from April. of 2008 to March. of 2019, to investigate the collective behavior of joint P/E ratio and price return. Focusing on the pricing of options, using the formalization of random walk to obtain an appropriate differential equation was studied for the probability function of price changing. This innovative statistical approach recognizes as the starting point for alternative models such as the Mandelbrot hypothesis and the Black and Scholes [16]. Furthermore, using the chaos theory to analyze the state of a financial system was studied by Chakraborti et al. [17] and Chen [18]. In these days, due to the strong correlations and subsequent entan-

glement of the markets, quantum mechanics can be applied as a suitable toolkit for studying the evolution of these entangled systems. A pioneer researcher in this area, was applied Quantum Mechanics into modeling some financial systems Khrennikov [19]. In a series of papers, were introduced a mathematical modeling based on Classical and Quantum Mechanics to investigate the dynamics of the financial systems Choustova [22, 23, 24]. They argued that the real financial conditions are include of hard as well as soft components. The previous ingredient may be governed by the classical Hamiltonian mechanics, while the recent is described by Bohmian quantum mechanics.
Using empirical data, was employed the quantum potential method to describe the mechanism of the fluctuations of price returns by Tahmasebi and his associates [13]. They found that creation the vertical potential walls could be responsible for this issue through the time entanglement of the price return. In addition, their findings expressed that the probability distribution function of the price return of the markets obeys a power law behaviour indicating a scale invariance of the price return, which, helps us to get information about the behaviour of the emerging and mature markets. Very recently, the classical as well as the quantum potential function, was estimated by Shen and Haven [14] using the empirical data for the commodity markets. They could confirm the creation of the potential walls and the scaling behaviour of the return variations. According to different information contents of Risk Information of Stock Market Using Quantum Potential Constraints and the classical quantum potentials, which show the hard and soft market conditions respectively, they pointed out the correlation between these two potentials. Osborne modeled the stock price trend using an issuance process and showed theoretically that the volume could affect the price variance. In this study, we are going to use bohmian quantum mechanics to find the trend of price return in the stock market and also predict the risk of each industry as well as find the relation between two important variables in the stock market which have never studied by mentioned model.

3 Theoretical Principles

With the exponential growth of program trading in the global financial industry, quantum finance and its underlying technologies have become one of the hottest topics in the finch community. Numerous financial institutions and fund houses around the world require computer professionals with a basic understanding of quantum finance to develop intelligent financial systems. In quantum mechanics and quantum field theory, wave function (ψ) is the most important component in the complete mathematical model, as it is the realization of wave-particle duality of quantum particles in this unique subatomic word of reality. Now, what is the corresponding wave function in quantum finance?
We can do it by measuring PDF (probability distributed function) of our data. Here in financial market we can obtain the pdf of a single variable or two variables and then calculate the quantum potential of that based on bohmian quantum potential formulas. By results which quantum potential charts give us, we able to analysing different markets. To study the financial markets and their trends, implementation of a quantum approach may prove conclusive. As such a sensible tool is the Bohmian quantum mechanics. In order to face the facts, the quantum potential is used for modelling real markets. The outcome would provide information on the expectations and limitations on the market under consideration. The probability distribution function of the variables which is represented by R is extracted from the market and substituted in eq. (1), where the quantum potential of that market in terms of its variable is gained. By having in hand information on the quantum potential based on the desired time scales, valuable information is obtained on the limitations put on the variables of that market. Hence, we carry out a case study on a typical market.

4 Methodology

In this step, we try to describe how Bohmian quantum mechanics helps us to understand the impact of different variables in the stock market. It is obvious that modelling a real stock market as a complex system, may not be performed by considering only a single variable of price return or P/E ratio. In addition, the existing evidence shows that different factors have their impacts on the behaviour of the (PDF) [3,18]. Given this, one may like to generalize the method adopted by Nasiri et al. [15] to a system of more than one variable. Two central equations of Bohmian quantum mechanics showing the impacts of the n dimensional systems are as follows:

$$\frac{\partial R^2}{\partial t}+\frac{1}{m}\sum_{i=1}^{n}\frac{\partial}{\partial qi}\left(R^2\frac{\partial S}{\partial q_i}\right)=0 \tag{1}$$

$$\frac{\partial S}{\partial t}+\frac{1}{2m}\sum_{i=1}^{n}\left(\frac{\partial S}{\partial qi}\right)^2+(v-\frac{\hbar^2}{2mR}\sum_{i=1}^{n}\frac{\partial^2 R}{\partial {q_i}^2})=0 \tag{2}$$

Which are gained by inserting the time-dependent wave function of n-independent Variables $(q_1.q_2.\ldots q_n.)$, i.e., $\psi(q_1.q_2.\ldots q_n.t)$ = $\mathrm{R}(q_1.q_2.\ldots q_n.t)$ exp (i$\frac{S(q_1.q_2.\ldots.q_n.t)}{\partial t}$),
in the Schrodinger equation
$i\hbar\frac{\partial\psi(q_1.q_2.\ldots.q_n.t)}{\partial t}=-\frac{\hbar^2}{2m}\sum_{i=1}^{n}\frac{\partial^2\psi(q_1.q_2.\ldots.q_n.t)}{\partial q_i^2}+V(q_1.q_2.\ldots q_n)\psi(q_1.q_2.\ldots q_n.t)$.
$\mathrm{R}(q_1.q_2.\ldots q_n.t)$ and $\mathrm{S}(q_1.q_2.\ldots q_n.t)$ are the amplitude and the phase of the wave function, and $\hbar$,q_i and m are the Planck constant, the ith component of the position, and the mass of the particle, respectively. In financial markets r would be the price return of a particular quantum financial market at time t, $\hbar$ representing the uncertainly of financial behaviour, m representing the intrinsic potential of financial markets such as the market capitalization of a particular financial product in financial market. In Eq. (3) in addition to the classical potential, $V(q_1.q_2.\ldots q_n)$, there is another potential:

$$U(q_1.q_2.\ldots q_n.t)=\frac{\hbar^2}{2mr}\sum_{i=1}^{n}\frac{\partial^2 R(q_1.q_2.\ldots.q_n.t)}{\partial q_i^2}=\sum_i^n U_i(q_1.q_2.\ldots q_n.t). \tag{3}$$

Which is recognized as the quantum potential for an n-dimensional system. Note that, if R in Eq. (3) is a separate function of n-independent variables, the corresponding quantum potential reduces to the sum of n one-dimensional quantum potentials. In this special case, as is shown in Fig. 3, 4, the domains of the variables are fixed and confined by the corresponding separable quantum potentials. However, as will be shown later, at least in our data this is not the case and the evidence does not always allow for the separation of the variable technique to solve the problem. This means that R, in general, is not a separable function of n-independent variables$(q_1.q_2.\ldots q_n)$.Nevertheless,one may still express the total quantum potential as the summation of n quantum potentialsU_i,i = 1,2,...,n as a function of $(q_1.q_2.\ldots q_n)$ family, governing q_i coupled with the remaining dependent group of variables of the family. Even, one may consider various cases intermediating the above extreme limits, where the group of n variables could be divided into independent subgroups of dependent variables. Corresponding to each independent variable in a given subgroup, one may define a quantum potential of partially coupled dependent variables. The selection of the best kernel function and bandwidth will enhance the results.

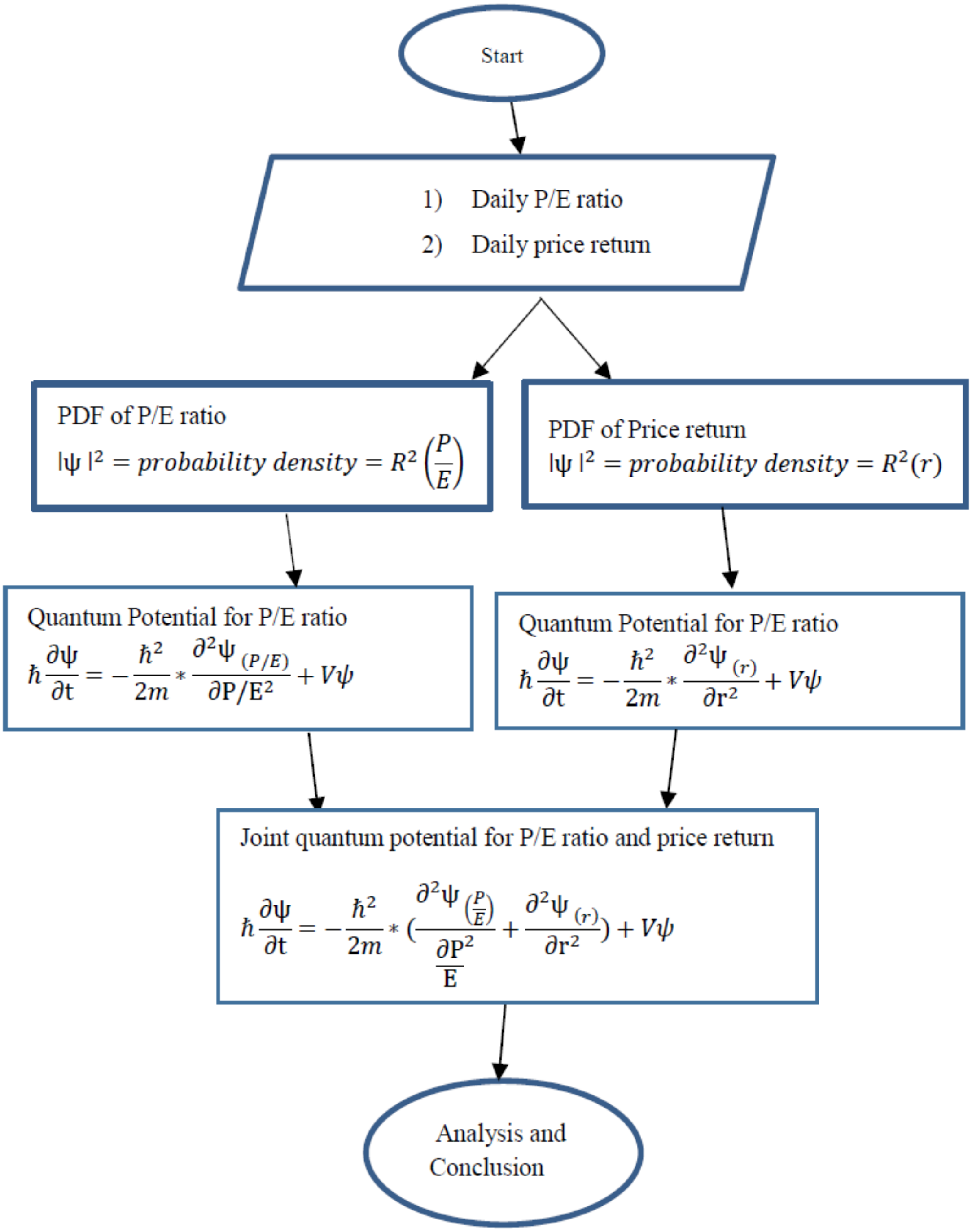

Fig. 1: Steps for obtaining the results

However, considering the complexity of such a selection, we do not test all of the possible functions, but simply use the Gaussian density as the kernel function. were calculated and we pointed out that the quantum potential causes restrictions in amplitude changes with respect to the time scales of the data, which is consistent with the results from other researchers [21]. we emphasized that the quantum potential can be a measure of the amplitude changes for return [21]. Therefore, there should be a very close link between the quantum potential and the volatility (that is, the financial risk) and we will discuss

this further in the following part. In detail, it can be seen that the distance between the quantum potential walls shows that the risk of those stocks with shorter distance are less than others. On the other hand, different companies may face different situations due to their economic conditions and the globalization of economic activities, such as industrial services. Therefore, the different distances between the potential walls, which are also a kind of measure of the amplitude variations of the price returns may not be completely explained by those “hard” economic conditions. In fact, the real financial market may be considered a complex cognitive system, which includes conditions regarding the behaviour of traders. In the quantum-like model, Choustova describes such conditions as “soft” or mental market conditions, which are included in the quantum potentials [24].
We realized that we would have less risk when bound of volatility is smaller and then the amplitude of the quantum potential reduces, on the contrary increasing in bound of volatility and expand the distance between the two potential walls will cause increasing in risk of stocks. In the present study being as practical as possible, empirical industries data are analysed and compared with the solutions extracted by our forward modelling based on the quantum Bohmian approach. We can find out to results follows: The quantum potential provides boundaries which confine the price return and P/E ratio or any variables fluctuations within their walls. In other words, by increasing the potential, the possibility of a variables decreases. These boundaries have a power law behaviour with time scale.These boundaries are also estimated analytically (eq. (2)). However, due to the fact that the walls are non-vertical, anything could happen, though with low probability. The distance between the walls which depends on the time scale is a measure for the amplitude variations of the variables (price return and P/E ratio in this study). forward modelling price or P/E ratio fluctuations of various markets or different industries would prove adequate for companies and individuals to act more rational on their finances. The Fig. 1 is shown the steps for obtaining the results (quantum potential) by detail.

5 Results and Discussions

As we mentioned in the previous section, the practical behaviour of real markets, recognized as the complicated dynamical systems, is expected to be affected by several variables. In other words, examining the evolution and outcomes of a market by a single variable model, may be an unreasonable/unreal approach to the problem and far from the reality. To answer the questions ‘why the price return and/or the P/E variations have never been experienced’, ‘What the joint PDF of these variables is’ and ‘whether these variables are inherently independent’, one needs to have information about the functional behaviour of the PDF in its general form. let us first consider the case of markets with a PDF as a function of a single variable, i.e., either the price return r, represented by R(r) and P/E ratio represented by R(P/E). In Fig. 2(a), R(r) for all possible values of returns during a time interval of April. 2008 to march. 2019 is plotted for the metal & chemical industries in Tehran exchange market. We extracted PDF for daily returns of these two markets. For the next step based on the second equation in flowchart we obtained the quantum potential charts using the pdf. See Fig. 2 and its caption, the quantum potentials of returns in metal industry are illustrated in respectively Fig. 2. By looking at the chart of potential walls we can see that since the walls are short any value even very small is probable. We observe that there are a pair of potential walls which prevent the returns from deviating too far away from the mean values. For the return variable in Fig 2 (Metal industry), it can change within the range from the value around −4 and +5, and breaking these walls is so hard and the resistances which we can see in potential well are maximum in these points. but it is possible by some events such as presenting financial statement for a fiscal year and having net income in it or announce that the company wants to pay profit to shareholders and… so, companies can cross from that. For example, we saw that we had a big decrease in price of stocks and therefore the return after announcing that the company do not increase the capital so far. We have some peaks in -2.5 and +4 as return but cutting these walls are more possible.

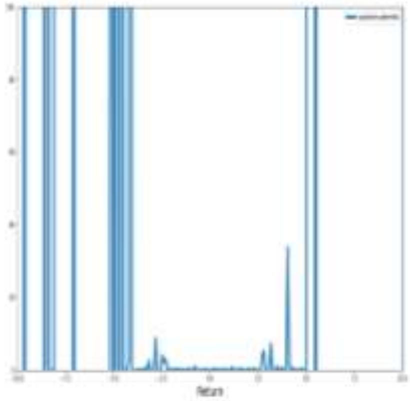

Fig. 2: Quantum potential for price returns in Metal industry

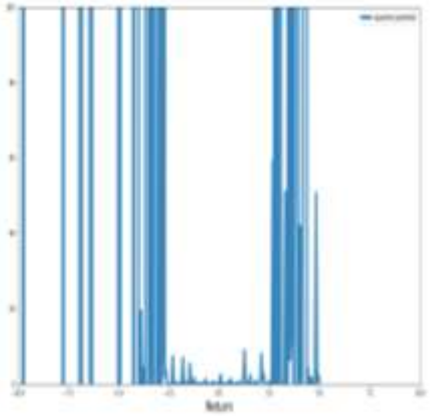

Fig. 3: Quantum potential for price returns in chemical industry

As we can see the walls in negative returns are stronger and more than positive return. See Fig. 3 (chemical market) and its caption, the quantum potentials of returns in chemical industry are illustrated in respectively Figs. 3 with h and m being equal to 1. We observe that there are a pair of potential walls which prevent the returns from deviating too far away from the mean values. For the return variable, it can change within the range from the value around +2.5 and -2.5, and breaking these walls is so hard and the resistances which we can see in potential walls are maximum in these points. We have some peaks between 2.5 till -2.5 as return but cutting these walls are easier than those high walls. As we can see the walls in negative and positive returns are the same. We focus only on the first high potential walls in each chart. By comparing two Figures with each other we can see that the potential walls in metal market allows variations for the price return in 9 percent (total negative and positive price return) but the potential walls in chemical market the variation are in 5 percent only therefore, the risk and also the return in Metal market are more than chemical market because the distance between two walls is longer in Metal market. We point out that the quantum potential causes restrictions for amplitude changes with respect to time scales of data. Therefore, we can say that those companies which have longer amplitude between their potential walls have higher risk than those companies with shorter amplitude, because different return amount in those companies are more possible. Figure 2, 3 show that the amplitude of metal market in longer than chemical market therefore the metal market shares have more risk than chemical market. See Fig.4 and its caption, it is illustrated that the amplitude of P/E in metal market is around 25 and we can predict that we wouldn't gain more than 25 as the P/E ratio in metal industry, also we have another wall with less power which shows that the P/E ratio in some specific points is more possible.

For the P/E variable in metal industry we can predict that it can change within the range from 2 to +25. See Fig.5 and its caption, it is illustrated that the amplitude of P/E ratio in chemical market is around 20 and we can predict that we wouldn't gain more than 20 as the P/E ratio in this industry, also we have another wall with less power which shows that the P/E ratio in some specific points is more possible.

Therefore, the amount of P/E ratio which we can predict in chemical industry is less than metal industry. For The P/E variable in chemical industry we can predict that it can change within the range from 2 to +20. We think that we should pay attention to this trend and pattern, so try to buy or sell in these picks or try to buy the shares of some companies which have less risk by comparing the amplitude of their potential walls.

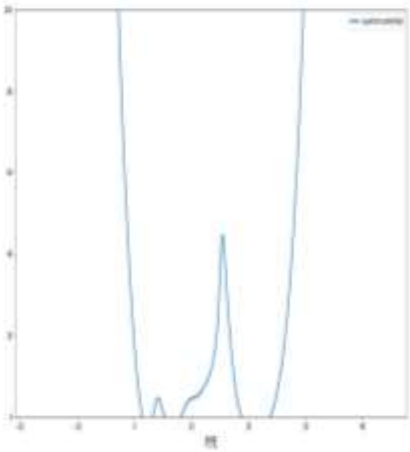

Fig. 4: Quantum potential for P/E in Metal industry

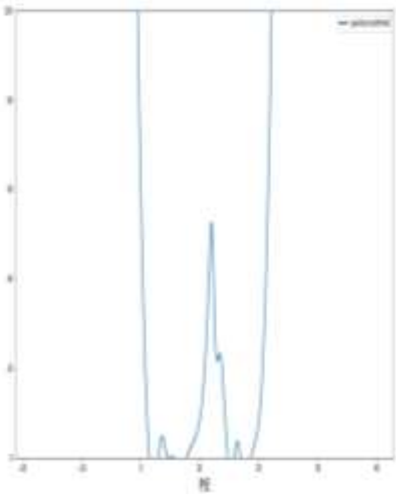

Fig. 5: Quantum potential for P/E in chemical industry

See Fig. 6 and its caption, the trace of intersections of a plane perpendicular to the vertical axis for different values of the PDF plane in metal market to obtain is probability contours. According to Figs.

6 and 7, it is obvious that the absolute value of price return and the P/E doesn't have any interaction with each other and no one cause any changing on another one. Therefore, r and P/E do not behave as two dependent variables. If they were, the traces shown in these Figures would irregular rather than rectangular shapes as it is. In other words, the two-dimensional PDF surface would become the surface of a pyramid with irregular. The dependence of r and P/E means that the P/E doesn't have its impact on price return and vice versa. Therefore, their behaviour is the same in the market and when one of them increases another one in increase too and vice versa. We can see some light points in Fig. 6 and 7 which show the points that we have more probable amount of price return and P/E ratio.

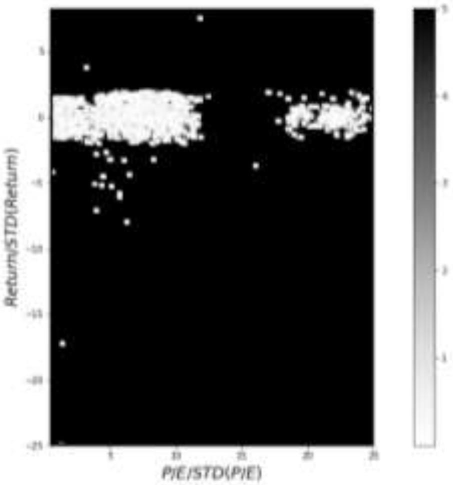

Fig. 6: The joint quantum potential in Metal market

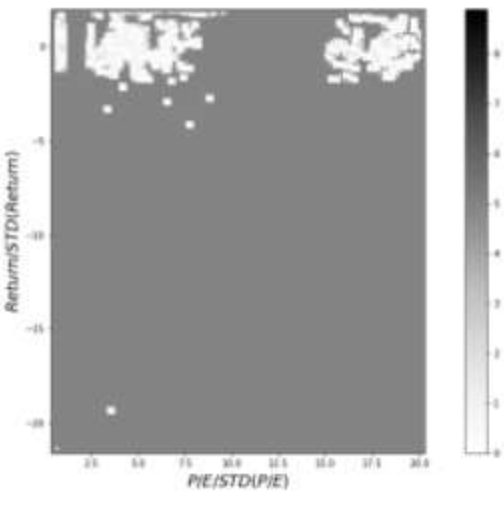

Fig. 7: The joint quantum potential in chemical market

See Fig. 7 and its caption, the trace of intersections of a plane perpendicular to the vertical axis for different values of the PDF is shown in (r,P/E) plane in chemical market to obtain is probability contours. According to Fig. 6, it is obvious that the absolute value of price return and the P/E have interaction with each other and one cause changing on another one. We can see some light points in Fig. 7 which show the points that we have more probable amount of price return and P/E ratio in this industry. we can see that the variation of values which are probable for chemical market are less than metal market because of the power potential walls which this industry has quantum potential walls for each variable as well. Therefore, both charts show the same results, and we can use this model as a scientific model with power theoretical framework instead of some experimental or statistical models which more use in Iran these days.

6 Conclusion and Suggestions for Future Studies

It is obvious that the exact modelling for evolution of a market, being considered as an extremely complicated dynamical system, could not be solely determined by some of variables. Too much soft (behavioural and psychological) and hard (economical) factors are present and have their own impact on the markets returns. This is the reason for generalizing the methodology as adopted by Nasiri et al. [15] to a multi-variable and complicated systems in Section 2. However, all that should not disappoint the researchers from trying to model the issue by a simplistic as well as scientific point of view. In spite of the fact that such modelling, with presumably maximum number of simplifications, does not consider all relevant ingredients, but can still detect the essential identity of the real problem without worrying about its full complexity. Although, as we referred before, some authors have investigated the effect of the price return together with the trading volume, most of the researchers have assumed the price return as a single variable describing the evolution of the markets which have been amassed in the Econophysics literature. In this paper, the behaviour of price return in the presence of P/E is investigated. The approach is an extension of the method used in another study by Nasiri et al. [15], who were inspired by two other prior studies [20,21]. The present results, consistent with what were previously reported for price return [19], show that there exists a quantum potential controlling the dynamics of the real market variables which confines its variations into a certain domain walls.

In addition, the joint probability distribution of return and P/E introduced here shows that these two variables are dependent and have interaction with each other. Another important finding reveals that behind the observed behaviour of the joint probability distribution function and the corresponding is probability contours, there exists a joint quantum potential due to the correlation between a price and a P/E and their prior-day price and P/E, respectively. Based on quantum potential chart one deals with price return or P/E quantum potentials, separately, as shown in Fig.2, 3 but in joint quantum potential chart one has robust return or P/E intervals confined by the corresponding fixed potential walls.

As explained in Section 2, with the use of the quantum potential method in studying stock markets, one may distinguish between four distinct cases. The first case deals with representing the markets by means of the quantum potential as a function of single independent variable, ignoring the impact of all other possible variables. In this case, the interval of variations of the variable is fixed and confined by the corresponding quantum potential. This method has been adopted and discussed for instance by Tahmasebi et al. [13], Shen and Haven [14] and very recently by Nasiri et al. [15], arguing that the quantum potential is a function of price return as a single variable with a fixed variation interval. In the second case, that is considered here, the markets are represented by the quantum potential as a function of two joint variables leading to a bidirectional causality relation between return and P/E ratio. Thirdly, the quantum potential of the markets is taken as a function of more than two variables without any subgroup structures and is argued to govern the fluctuations of each variable through the impact of all remaining groups of variables. Finally, the quantum potential is again taken as a function of more than two variables; however, in contrast to the former case, here the variables could be categorized into different subgroup structures. In this case, the quantum potential, when obtained, could shed light on deeper layers of the corresponding markets. Further, in this study we considered two different industries in one market as isolated dynamical system without considering the impacts of other existing markets. However, taking into account the globalization of stock markets, such a simplified model could not explain the resulting complicated situation. Thus, one may investigate the interconnections of different correlated stock markets by introducing an appropriate quantum potential based on the markets' quantum interference.

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