An Analysis of the Repeated Financial Earthquakes

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ABSTRACT

Since the seismic behaviour of the earth’s energy (which follows from the power law distribution) can be similarly seen in the energy realized by the stock markets, in this paper we consider a statistical study for comparing the financial crises and the earthquakes. For this end, the TP statistic, proposed by Pisarenko et al. [37], is employed for estimating the critical point or the lower threshold, i.e. the point beyond that the market energy follows from the power law (Pareto) distribution. The results confirm the deviation of the energy from the Pareto distribution in the high quantiles of the energy data. The upper threshold that the energy's distribution is changed from the Pareto to another distribution is also estimated by TP statistic. A simulation study is employed for checking out the statistical behaviour of the estimated thresholds. Finally, the magnitude of the financial earthquakes is studied. The results indicate that the domestic and the international events especially the global financial crisis have caused the financial earthquakes in Tehran Stock Exchange. Also, the positive relation between the daily energy released and the daily magnitude of the shocks that was connected by Gutenberg and Richter [31] is confirmed.

1 Introduction

Financial crisis has been one of the considerable characteristics in economic perspective [1]. The crises before the 19th century, such as bubble collapse of tulip mania, to the crises in the twentieth and twenty first centuries, affirm the repeatability of crises. By studying the history of crises, it can be said that a large number of crises had occurred in the economy of the United States. The economy of U.S. in the period 2007-2009 faced a credit crisis that turned into a financial crisis and after entering to the U.S. stock market it was spread in many countries. It disturbed the performance of the actual and financial sectors in different countries so that some economies were pushed one step and some several steps back. Financial crises have influenced countries differently due to their type of interactions and links to global economy and the political responses of governments. On the one hand, whatever the degree of the interaction and integration between the financial markets has been more; the effects of the crises were identified via the financial markets and private capital flow. On the other, whatever the financial interactions have been less, the effects of crises were mostly observed via the international trade and foreign exchange. In this regard, the suppliers of energy and raw material are more affected by crises, because invest-
ment and economic activities descend in a number of oil-dependent developing countries due to the decrease of oil price [2]. Thus, the international shock transmission from developed countries to the others, developed and developing ones cannot be considered as zero. These shocks lead to the volatility behaviour of the macroeconomic variables in turn.

This volatility behaviour in duration of the financial crisis can be described as the earth's seismic activity in an earthquake. Just like, the shake of the earth is in the nature of it and exists continuously; crisis is the inseparable part of economic cycle which has a historical background. Crises, similar to an earthquake, can make turbulence in markets and its seismic waves can spread out from focus.

Some researchers, such as Potirakis et al., have analyzed earthquake dynamics and financial systems in the mathematical frameworks [3]. Negrea also believes that financial crises, like earthquakes, can be explained as a “self-organized criticality”. Based on the power law, a stock market, as a dynamic system, can reach to a critical point (to a financial crash) and even go ahead up to the infinity [4]. Hence, an understanding of the dynamic response of the stock market to the domestic and foreign crises is very important. In this paper, the kinetic energy released by Tehran's stock market is computed. Then, the occurrence of the financial earthquakes is identified and their magnitude is calculated. Also, a simulation study employing the parametric bootstrap is conducted in order to check out the statistical behaviour of the estimated thresholds.

2 Literature Review and Previous Studies

Most of the financial crises in developing countries are originated from the large financial centers in developed countries. They gradually sacrifice developing countries. According to the study of the international monetary fund, during the recent financial crisis, the economic growth reduction in developing countries is less than the developed ones. In this study, it is expressed that the economy of countries damages severely from three main shocks, financial turmoil (which confines access to foreign funds), demand reduction in developed countries and decrease of goods price, especially oil and energy [5]. In this regard, Naude divided the transmission channels of the financial crisis to developing countries into 3 sections: 1) banking failures and reductions in domestic lending, 2) reduction of export earnings, and 3) reduction in financial flow to developing countries. He believes that the first channel, due to the little relation between the banks of the developing countries and the international financial system, will have relatively limited effects. The second channel impacts on the exporting countries of the commodities and raw materials. The third one usually impacts on countries which are dependent on foreign financial flows [6].

According to the study of IMF, the global financial crisis influences on developing countries through two channels: 1) financial contagion and 2) economic downturn that its channels consist trade and trade prices, remittances, foreign direct investment and equity investment, commercial lending, aid, and other official flows [7]. Shikimi and Yamadab have introduced trade and financial channels. They find that while the trade channel temporarily influences the financing behavior of Asian firms exporting to the U.S., this channel has no effect on cash-holding and investment behavior. Also, the financial channel has effect on the financing behavior of Asian firms with less available outside financing [8].

Financial crisis has transmitted from one market to another and from one country to the other ones and finally encompasses the world economy. King and Wadhani studied the contagion effects between equity markets after the crash of 1987. They believe that a "mistake" in one market such as the
crash is transmitted to the others. Evidence shows the increased correlation among equity markets after crash [9]. Similar results are found in the paper of Bertero and Mayer among the equity markets of 23 industrialized and developing countries [10]. The transmission of shocks among American markets and foreign ones has been analyzed by Samarakoon. The results indicate the bi-directional, yet asymmetric, interdependence and contagion in emerging markets [11]. Boubaker et al. in their study over both the pre- and the post-subprime crisis periods have presented statistically significant results from the effects of contagion between the U.S. equity market and the equity market of developing and emerging countries after the global financial crisis [12]. In a comparative study on the economic development process of 5 groups of countries during the 2007-2009 financial crisis, Long et al. found out that developed countries and European ones are influenced relatively more and Asian emerging market countries are influenced relatively less [13]. Yamamoto has represented that the influence of financial shocks is greater than commercial shocks in 2007-2009 and financial linkages is strong among U.S. and Asian economies, exclusively for Asian developed economies [14]. Saleem has investigated the international relationship between the Russian equity market and global market. The results point out that there is some evidence on contagion at the time of the financial crisis, although this relation is weak [15]. Magheryeh et al. have concluded that the stock markets of MENA countries are approximately correlated and, in a normal condition, are integrated to the US market weakly. This structure of relations changed in 2008. In stress, information is broadcasted from the United States to the other countries and MENA markets are integrated more. Moreover, in stress, there is a unilateral relationship from U.S. market to other ones [16]. Kim et al. have pointed out that the spillover effects of the financial crisis in 2008 on Asian emerging countries is a short-lived and non-negligible. Besides, the spillover effects are much stronger in foreign currency market than equity market and foreign investment plays an undeniable role in conditional correlations in the international equity markets [17]. Neaime has showed that the degree of vulnerability of the equity markets that are financially integrated to the global equity market is more from the local and international financial crises. This vulnerability is caused by the weak regional integration on the one hand and the strong financial integration with developed economies on the other [18]. Ghasemi and Sarlak have surveyed the impact of the financial crisis on conservative accounting and transparency of banks in Iran. The results of testing the hypotheses show that the crisis on conservative accounting and transparency of banks has a significant impact [19]. Ahmadi and Kordloei have studied the effect of financial distress on the investment behavior of companies listed on Tehran Stock Exchange. The results show that firms with less investment opportunities tend to be less likely to invest. Also, distressed financially firms with more investment opportunities are more likely to increase investment [20]. Cardak et al. have investigated the effects of the global financial crisis on the stock holding decisions of Australian households. The results indicate evidence that financial crises cause households to become more myopic, increase their responsiveness to shocks. In addition, they focus more on past extreme returns [21]. BenMim and BenSaïd have studied financial contagion across stock markets during crisis and tranquility periods. The results indicate a significant change in the connectedness and shock transmissions during both periods. Also, they find strong evidence of financial contagion with the Eurozone at its origin [22].

In this regards, several studies have detected similarities between the financial market crises and the earthquakes. Johansen and Sornette have studied the stock market crashes as outlier. The results imply
that great crashes are caused by processes which may lead to visible pre-cursory signatures [23]. Selcuk has conducted a study on financial earthquakes, aftershocks and scaling in emerging stock markets. The findings indicate that the empirical scalar rule is a power law. Besides, after a decline in returns, the market fluctuations above a certain threshold demonstrate the power law decay [24]. In the statistical physics framework, Potirakis et al. have pointed out a dynamic analogy between earthquakes and economic crises [3]. Gresnigt et al. have suggested a modeling framework to predict probability of crash in the stock market. They found considerable similarities between stock returns during a financial market crash and seismic activities during an earthquake. They have developed an Early Warning System for future crash. Their modeling framework can make the crash probability predictions in the medium term [25]. Jagielski et al. have studied theory of earthquakes inter-event times applied to financial markets. They show that the formalism of the Hawkes process used for earthquakes can successfully model the PDF of inter-event times between successive market losses [26]. Ikeda has investigated multi-fractal structures for the Russian stock price returns via the multi-fractal detrended fluctuation analysis (MFDFA). The results show that the multi-fractality degree of the Russian stock market can be ordered as one of the developing countries [27].

Analogous to Richter scale for an earthquake, Zumbach et al. have introduced the Scale of market Shocks (SMS) indexes as a scale of shocks in the foreign exchange market to determine the size of shocks which is based on the price volatility. These indexes are created assuming that market releases a kinetic energy [28]. Kapopoulos and Siokis have pointed out that aftershocks in the emerging and developed stock markets follow the Gutenberg–Richter law in geophysics [29]. Siokis has presented the distribution of magnitude of the stock market shocks, and modeled the dynamics of the stock market index based on the Gutenberg–Richter law. The obtained results point out that the distribution of market volatility pre– and post–crash is characterized by the Gutenberg–Richter law and the rate of the decay of the aftershock sequence is expressed by the Omori law [30]. Negrea has presented the indicator of the financial crisis magnitude by the Gutenberg–Richter relation and computed it for Dow Jones and S&P 500 index data. Also, he determined the financial crashes hierarchy in different periods [4].

3 Methodology and Models

Since the results of this research can be used in the policy-making process, this research based on the purpose is an applied study and based on the method is analytical-descriptive. In this regard, according to theoretical foundations and to achieve the research objectives, the three hypotheses are presented:

i) A financial earthquake has ever occurred in Tehran stock market.


iii) There is positive relation between the daily energy released and the daily magnitude of the earthquake that was connected by Gutenberg and Richter [31].

The statistical sample of this research is the Tehran Stock Exchange in the daily period of 1997-2016. The information gathered from the Tehran Stock Exchange website using library method. To data analysis, statistical methods is also applied by the R software, version 3.3.2.

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1 The data are available in URL: http://www.tse.ir/archive.html
3.1 Proposed Model

In this subsection, a comparison between the financial crisis in economy and the earthquake in seismology has been considered by using the power law distribution. Most of such natural phenomena follow from the power law distribution instead of the normal distribution [32]. The power law distribution is an example of a heavy tail distribution. The pure power law distribution was introduced by Pareto and then called the Pareto distribution. There are some common definitions for the Pareto distribution, but here we use the one which is more prevalent in earthquake studies and is introduced as the Gutenberg–Richter relationship. A random variable $E$ is said to have the Pareto distribution, if it has the following cumulative distribution function (cdf)

$$F(E_t) = 1 - \left(\frac{E_c}{E_t}\right)^\theta, \quad E_t > E_c$$  \hspace{1cm} (1)

where $E_c (>0)$ as a scale parameter is the representative of the critical point and $\theta (>0)$ as a shape parameter is called tail index or power index. Moreover, suppose that $n_{ig}$ is the frequency of the values in the interval $\log u \pm \delta(\log u)$ and $u$ is a measure of size, then we have

$$\log n_{ig} = a - bM,$$  \hspace{1cm} (2)

where $a$ and $b$ are constant. Replacing $\log u$ with $M$ in Eq. (2) yields the Gutenberg–Richter relationship, i.e.

$$\log n_{ig} = a - bM$$  \hspace{1cm} (3)

It is obviously observed that the Gutenberg–Richter law is a power law that relates the number of earthquakes to their magnitude. Some studies consider the Gutenberg–Richter law as the best description of the magnitude of an earthquake [33, 34]. Also, according to the study of Gutenberg and Richter (1956), the linear magnitude–energy relationship for the time series data can be written as

$$\log_{10}E_t = a + bM_t,$$  \hspace{1cm} (4)

where the threshold energy is represented by the parameter $a$, i.e. $E_c = e^a$, which corresponds to $M_t = 0$ [31]. Thus, Eq. (4) can be recast as

$$M_t = \frac{1}{b} \log_{10} \left(\frac{E_t}{E_c}\right)$$  \hspace{1cm} (5)

If $E_t = E_c$, the magnitude of an earthquake is zero and if $E_t > E_c$, an earthquake happens [4]. According to the power law, the earth energy can be related to the probability of the occurrence of an earthquake having energy $E_t$ as

$$P(E) \cong C \left(\frac{E_c}{E_t}\right)^\theta$$  \hspace{1cm} (6)

where $C, E_c$ and $\theta$ are some constants as those defined in Eq. (1). The probability (6) is the body of the survival function of the Pareto distribution with cdf (1). This fact is the reason that the seismologists express that seismic energy tends to follow a Pareto distribution. Therefore, combining, Eq.s
(1), (5) and (6) gives the magnitude under the Pareto distribution as
\[ M_t \simeq \frac{1}{\theta} \ln \left( \frac{1}{1 - F(E_t)} \right) + \nu \]  
(7)
where the right hand side (excluding the constant \( \nu \)) is the logarithm of the quantile function of the Pareto distribution with the unit scale parameter.

For studying the similarity between the occurrence of an earthquake and the occurrence of a financial crash, we employ the probability distributions. As the energy released during an earthquake follows the Pareto distribution [35], the financial market can also reach to a critical point (threshold) according to the power law. Therefore, we need to detect the critical value of the energy released by the market that is named the critical financial market energy (threshold financial market energy). The critical energy or threshold energy is described as the energy beyond that an earthquake (financial crises) happens. Then, the magnitude of financial earthquakes is calculated based on the Richter scale.

### 3.2 The Kinetic Energy of the Stock Market

In physics, the kinetic energy is the energy which exists in a moving body and causes its motion. In economy, also the stock market can be considered as a moving body. The kinetic energy of a moving body \( E_k \) is dependent on its mass \( m \) and velocity \( v \), more specifically
\[ E_k = \frac{1}{2} mv^2 \]  
(8)

To calculate the kinetic energy of a market, the quantity of shares traded can be substituted for mass. As the velocity is the instantaneous change of the moving body positions \( dx \) over the instantaneous change of time \( dt \), \( v = dx/dt \), so the difference of the value of the market price index in two consecutive trading days is defined as velocity because the trading volume has a great impact on the direction of share movements and is related to price index. Therefore, the daily kinetic energy can be obtained as
\[ E_t = \frac{1}{2} m_t \left( \frac{dV_t}{dt} \right)^2 \]  
(9)
where \( m_t \) and \( V_t \) are the daily trading volume of the market and the daily value of the market price index, respectively. It is noticeable that the relative changes are better than the absolute changes. So Eq. (9) is recast as
\[ E_t = \frac{1}{2} \ln m_t \left( \frac{d\ln V_t}{dt} \right)^2 \]  
(10)

Since the trading volume is continually changing, the market energy in a trading day can be different from the other ones. As a result, for comparing the amplitudes of the seismic waves that are the waves of energy, the magnitude scale is applied.

In order to measure the daily magnitude of the financial earthquake, it is necessary to choose the most appropriate probability distribution for the stock market energy as the first step. Then, the critical energy (or threshold), \( E_c \), is estimated by using a non–parametric method, as the point just after that the data follows from the Pareto distribution. In the third step, if a deviation from the Pareto distribution is observed, then, a more suitable probability distribution should be fitted to the rest of the market.
energy data. Finally, the magnitude of the financial earthquakes is calculated by using the quantiles of the mentioned distributions [4].

4 Analysis and Findings
4.1 Preliminary Analysis
We have surveyed the daily kinetic energy of Tehran Stock Exchange from October 5, 1997 to September 5, 2016. This market energy has been obtained based on the daily Tehran price index (TEPIX) and the total daily trading volume using Eq. (10). Table 1 shows the descriptive statistics of the energy. Also, the trend plot of the data is plotted in Fig. 1.

![Fig. 1: The daily energy](image)

<table>
<thead>
<tr>
<th>Statistic</th>
<th>value</th>
<th>Statistic</th>
<th>value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>3.58e-04</td>
<td>Q1</td>
<td>1e-05</td>
</tr>
<tr>
<td>Median</td>
<td>5.12e-05</td>
<td>Q3</td>
<td>2.4e-04</td>
</tr>
<tr>
<td>Min</td>
<td>1.01e-12</td>
<td>Skewness</td>
<td>13.409</td>
</tr>
<tr>
<td>Max</td>
<td>0.0342</td>
<td>Kurtosis</td>
<td>268.501</td>
</tr>
</tbody>
</table>

Note: Q is an abbreviation for the sample quartile. The length of the data is $n = 4371$.

From Table 1, it is observed that the market energy does not follow a normal distribution, which is clearly confirmed by a common normality test as well. The mean of the energy is even more than Q3, as a result, the distribution of the data belongs to the class of fat–tailed distributions.

4.2 The Appropriate Distribution Fitting
For determining the best fit on the data, several continuous distributions have been fitted on the data using the maximum likelihood estimation (MLE) method. Then, Kolmogorov–Smirnov (KS) test has been employed for checking the goodness–of–fit. Finally, for comparing the various models, we have considered the well–known Akaike Information Criterion (AIC) defined as $AIC = -2\ln L + 2p$,

where $L$ and $p$, are the maximum value of the likelihood function and the number of the estimated parameters, respectively. The best fitted distribution has the minimum AIC. The computations have been done by the R software, version 3.3.2, and employing the routines 'optim', 'DEoptim' (in the R
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package DEoptim) and 'ks.test'. Table 2 shows the first 9 fitted distributions having KS–pvalue greater than 0.05 as well as the lowest AIC among the other candidates.

Table 2: The fitted distributions on the daily energy of Tehran Stock Exchange

<table>
<thead>
<tr>
<th>Distribution</th>
<th>AIC</th>
<th>Rank</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pearson type VI</td>
<td>-71161</td>
<td>1</td>
</tr>
<tr>
<td>Dagum</td>
<td>-71159</td>
<td>2</td>
</tr>
<tr>
<td>Weibull</td>
<td>-70974</td>
<td>3</td>
</tr>
<tr>
<td>Burr</td>
<td>-70966</td>
<td>4</td>
</tr>
<tr>
<td>Gamma</td>
<td>-70819</td>
<td>5</td>
</tr>
<tr>
<td>Log-Logistic</td>
<td>-69999</td>
<td>6</td>
</tr>
<tr>
<td>Pareto</td>
<td>-67677</td>
<td>7</td>
</tr>
<tr>
<td>Frechet</td>
<td>-65059</td>
<td>8</td>
</tr>
<tr>
<td>GEV</td>
<td>-63752</td>
<td>9</td>
</tr>
</tbody>
</table>

Institution shows that Pearson type VI distribution with the probability density function (pdf)

\[ f(x; \theta) = \frac{\Gamma(\alpha_1 + \alpha_2)}{\beta \Gamma(\alpha_1) \Gamma(\alpha_2)} \left( \frac{x}{\beta} \right)^{\alpha_1 - 1} \left( 1 + \frac{x}{\beta} \right)^{-\alpha_1 - \alpha_2}, \quad x > 0, \quad \theta = (\alpha_1, \alpha_2, \beta), \quad \alpha_1, \alpha_2, \beta > 0 \]  

(11)

and the estimated parameters \( \hat{\theta} = (0.306, 1.696, 0.0009) \), has the best fit on the energy data, since it has the lowest AIC and the best behavior in QQ–plot comparing to the other competitors (\( \Gamma(\cdot) \) in Eq. (11) stands for the complete gamma function). The corresponding fitted curve, the empirical cdf (ecdf) and the QQ–plot of Pearson type VI distribution are presented in Fig. 2.

4.3 The Threshold(s) of the Stock Market Energy

As it is observed from Fig. 2, the daily energy data deviates from Pearson type VI distribution in the upper quantiles. Therefore, we must find a more suitable distribution for the tail of the data. Newman (2005) believes that "a point at which the length–scale in a system diverges is called a critical point or a phase transition. Also, critical phenomena are known as things occur in the vicinity of critical point, of which power–law distributions are one example" [36]. In addition, Negrea [4] has defined a critical point as the point beyond that the explosion of the financial energy occurs. If the market energy is higher than the critical value of the energy, the stock market crash happens. The burst of energy in a stock market behaves as the explosion of seismic energy during an earthquake and they both follow the power law distribution (Pareto distribution) [4]. The Gutenberg–Richter law in terms of seismic moments coincides to the Pareto distribution. The results show that all daily energy data higher than the critical point \( E_c \) follow from the Pareto distribution. For estimating the critical point \( E_c \), a non–parametric approach proposed by Pisarenko et al., (2004), is applied. They have defined a statistic called TP that here we define it as a different form [37].

Definition 1. Consider the finite sample \( E_1, \ldots, E_n \) as a stock market energy data. Also, suppose that \( E_{(1)} \leq \cdots \leq E_{(n)} \) denotes the sorted sample in the magnitude order. Then, the TP statistic of the observation \( E_{(t)} \) is defined as

\[ \text{TP}(E_{(t)}) = \frac{1}{n} \sum_{i=1}^{t} \frac{i}{n} \]
\[ TP_t = \left[ \frac{1}{n-t} \sum_{i=t}^{n} \ln \left( \frac{E(i)}{F(E(t))} \right) \right]^2 - \frac{1}{2(n-t)} \sum_{i=t}^{n} \left[ \ln \left( \frac{E(i)}{F(E(t))} \right) \right]^2, \quad t = 1, ..., n-1 \tag{12} \]

The estimated value of the parameter \( E_C \), denoted by \( \hat{E}_C \), is the observation \( E(t) \) that has the closest TP to zero among the others. Then, the sample \( E_1, ..., E_n \) has the Pareto distribution from \( \hat{E}_C \) onward. The TP statistics of all values of the daily energy of Tehran Stock Exchange have been calculated based on Definition 1. The observation having the lowest \(|TP|\) corresponds to \( \hat{E}_C = 0.0025 \) (See it in Fig. 1 and 2). In order to ensure that the all data higher than \( \hat{E}_C \) follow from the Pareto distribution, KS test has been used and the results are reported in Table 3.

**Table 3**: The MLE of the shape parameter of the Pareto distribution and KS test

<table>
<thead>
<tr>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \hat{\theta} )</td>
</tr>
<tr>
<td>Std. of ( \hat{\theta} )</td>
</tr>
<tr>
<td>KS statistic</td>
</tr>
<tr>
<td>KS p–value</td>
</tr>
</tbody>
</table>

The entries of Table 3 confirm once again that the energy follows from the Pareto distribution from \( \hat{E}_C \) on. However, according to all calculated TP statistics, a deviation from the Pareto distribution is clear.
so that the second threshold \( \tilde{E}_f \) may be needed. Hence, as mentioned earlier, based on the Gutenberg–Richter relationship, it is better to say that the Pareto distribution is suitable only for the data fallen into the interval \((\tilde{E}_c, \tilde{E}_f)\) and the values greater than \( \tilde{E}_f \) follow from another distribution which should be detected as well. Thus, in order to find a suitable distribution for the rest of the data, some distributions have been fitted on it. The results suggest the Wakeby distribution among the others. It is to be noted that although there is no closed form for the pdf of Wakeby distribution, its quantile function is

\[
Q(u) = \xi + \frac{\lambda}{\beta} [1 - (1 - u)\beta] - \frac{\gamma}{\delta} [1 - (1 - u)^{-\delta}], \quad 0 < u < 1
\]  

where the parameters \( \beta, \gamma \) and \( \delta \) are the shape parameters and the parameters \( \xi \) and \( \lambda \) are the location parameters. Also, \( \lambda \neq 0 \) or \( \gamma \neq 0 \), \( \beta + \delta > 0 \) or \( \beta = \gamma = \delta = 0 \), if \( \lambda = 0 \) then \( \delta = 0 \), if \( \gamma = 0 \) then \( \delta = 0 \), if \( \gamma \geq 0 \) then \( \lambda + \gamma \geq 0 \), are the imposed conditions. Domain is such that \( \xi \leq Q(u) < \infty \) if \( \delta \geq 0 \) and \( \gamma > 0 \), and \( \xi \leq Q(u) \leq \xi + \frac{\lambda}{\beta} - \frac{\gamma}{\delta} \) if \( \delta < 0 \) or \( \gamma = 0 \). The Wakeby distribution is one of the most usable distributions for analyzing the extreme events such as flood. In the other words, such a market involved in a deep crisis can be resembled to a flooded area. Hence, the second threshold energy, \( \tilde{E}_f \), is called the flood threshold energy. Actually, the TP statistic is deviated from zero for a non–Pareto distribution, so the observation corresponds to the highest absolute values of TP can be employed as an estimation of the flood threshold energy, \( \tilde{E}_f \). Based on this approach, we have \( \tilde{E}_f = 0.0078 \) (See it in Fig. 1 and 2). Table 4 also shows that the Wakeby distribution can be used as the parent distribution of the data higher than \( \tilde{E}_f \).

<table>
<thead>
<tr>
<th>( \xi )</th>
<th>( \lambda )</th>
<th>( \beta )</th>
<th>( \gamma = \delta = 0 )</th>
<th>KS statistic</th>
<th>KS p–value</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.007</td>
<td>0.011</td>
<td>0.101</td>
<td>0</td>
<td>0.133</td>
<td>0.94</td>
</tr>
</tbody>
</table>

It should be noted that the parameters of the Wakeby distribution have been estimated via the L–moment method, since there is no closed form for its pdf and so obtaining MLE is not feasible. To this end, the R package ‘lmomco’ has been used.

### 4.4 A Simulation Study

A bootstrap simulation is conducted in this section for analyzing the behavior of the estimated critical points \( \tilde{E}_c \) and \( \tilde{E}_f \). Since, the parent distribution of the energy has been determined in Section 4.2, we employ the ‘parametric’ bootstrap method. More specifically, \( B=1000 \) samples of size \( n = 4371 \) is drawn from the Pearson type VI distribution with pdf \( f(x; \Theta) \) given by Eq. (11). Then, based on the mentioned approach in Section 4.3, the thresholds \( \tilde{E}_c \) and \( \tilde{E}_f \) are computed for each sample. Thus, we have \( B \) observations for each of them as \( \tilde{E}_c = (\tilde{E}_c^{(1)}, \ldots, \tilde{E}_c^{(B)}) \) and \( \tilde{E}_f = (\tilde{E}_f^{(1)}, \ldots, \tilde{E}_f^{(B)}) \). Moreover, the corresponding TP statistics of \( \tilde{E}_c \) and \( \tilde{E}_f \), respectively denoted by \( TP_c \) and \( TP_f \), are calculated. The kernel density estimation (KDE) of the mentioned four vectors including their corresponding values obtained from the real data (dotted red lines), have been plotted in Fig. 3 and 4. Some of the points are observed from these figures are as follows.
As it was expected, the observations are distributed around the red lines. Density of $T_P$ is symmetric and highly concentrated about zero, while density of $T_F$ is asymmetric with hole at zero. Both of densities $E_c$ and $E_f$ are asymmetric and right-skewed.

Fig. 3: The KDEs of $E_c$ (left panel) and $T_P$ (right panel).

The %95 percentile bootstrap confidence intervals (PBCI) for the parameters $E_c$ and $E_f$ have been obtained of the form $(\hat{E}_c,0.025,\hat{E}_c,0.975)$ and $(\hat{E}_f,0.025,\hat{E}_f,0.975)$, where $\hat{E}_{c,a}$ and $\hat{E}_{f,a}$ denote the $a$th sample quantiles of $E_c$ and $E_f$. These intervals as well as the exact standard deviation (Std.) of the TP statistics obtained based on Pisarenko et al. [37], the bootstrap Std. as well as the bootstrap estimation of the corresponding TP statistics of $\hat{E}_c$ and $\hat{E}_f$ are reported by Table 5.

It can be seen that PBCIs contain the thresholds obtained from the data itself. In the insurance studies and the crisis management of the natural disasters, the reliable estimates of the magnitude of the natural disasters are especially attractive. In the rest of this paper, the magnitude of the earthquakes is estimated in order to investigate the magnitude of the crises in Tehran Stock Exchange.
Table 5: The thresholds of the energy with the corresponding TP statistics

<table>
<thead>
<tr>
<th>Threshold energy</th>
<th>$E_c$</th>
<th>$E_f$</th>
</tr>
</thead>
<tbody>
<tr>
<td>95% PBCI</td>
<td>0.0025</td>
<td>0.0078</td>
</tr>
<tr>
<td>TP statistic</td>
<td>0.0011-0.0313</td>
<td>0.0026-0.0315</td>
</tr>
<tr>
<td>Std. of TP</td>
<td>2.0114e-05</td>
<td>0.1082</td>
</tr>
<tr>
<td>Bootstrap Std. of TP</td>
<td>0.0175</td>
<td>0.0074</td>
</tr>
<tr>
<td>Bootstrap estimation of TP</td>
<td>0.0024</td>
<td>0.1646</td>
</tr>
<tr>
<td></td>
<td>5.9137e-05</td>
<td>0.0871</td>
</tr>
</tbody>
</table>

4.5 The Magnitude of the earthquakes

There is much continuous seismic activity in the earth and much persistent stress and turbulence in the stock market [38]. So, a scale is necessary to measure the state of the earth and the market as well. Richter [39] developed this scale. According to it, the magnitude of an earthquake can be obtained by the logarithm of the intensity\(^2\) of an earthquake (calculated by the amplitude of waves that seismograph recorded) to the intensity of a standard earthquake. Since the intensities of an earthquake follow a power law, it is written in the logarithmic form. As mentioned earlier, Gutenberg and Richter [31] have pointed the connection between the wave magnitude and the energy released by an earthquake. In the economic, some researchers such as Zumbach and et al. [28] and Maillet and Michel [40] constructed indices in analogy with the Richter scale. Recently, Negrea [4] has introduced a magnitude indicator according to the Gutenberg and Richter relation that we use it in this section.

If the energy follows the power law distribution at the time of occurrence of an earthquake, the magnitude is obtained by Eq. (7). However, since Pareto and Wakeby distributions determined as the statistical patterns of the financial crises, we have two threshold energies. When energy is higher than the critical energy ($\hat{E}_c$), the earthquake occurs in stock market and when energy released by market is higher than the flood energy ($\hat{E}_f$), the market is flooded. Thus, for measuring the magnitude of the financial crisis, we need the quantile function of the Wakeby distribution given in Eq. (13) as well. Then, a relation can be made between the logarithm of the market energy and the magnitude of an earthquake by a linear combination of the quantile functions as

$$ M_t = \begin{cases} \frac{1}{\theta} \ln \left( \frac{1}{1 - F(E_t)} \right) & \hat{E}_c \leq E_t \leq \hat{E}_f \\ \ln \left[ \frac{\xi}{\hat{E}_c} + \frac{\hat{\lambda}}{\hat{\beta} \hat{E}_c} \left( 1 - (1 - u_t)^{\hat{\beta}} \right) - \frac{\gamma}{\hat{\delta} \hat{E}_c} \left( 1 - (1 - u_t)^{-\hat{\delta}} \right) \right] & \hat{E}_f < E_t \end{cases} $$

(14)

where $u_t$ is the empirical cdf of the energy data evaluated at $E_t$. In the current data, there are 127 business days which their financial energy is higher than $\hat{E}_c$, i.e. $\hat{E}_c \leq E_t$. The daily magnitude of these points is computed using Eq. (14) and the results are plotted in Fig. 5. It is observed that despite the occurrence of earthquakes in during 1998-2016, the stock market is faced on the deep crises in the years 2003, 2008, 2009, 2013, 2014, 2015 and 2016 because the energy in this years is greater than or equal to $\hat{E}_f$. In addition, the relation between the energy released by market and the daily magnitude of the shocks is shown in Fig. 6. As it is expected, magnitude is an increasing function of the energy

\(^2\) The intensity is proportional to the square of its amplitude.
Also, the functional shape of $M_t$ and $E_t$ in Fig. 6 confirms the empirical magnitude-energy relation given by Eq. (4). The red points in both Fig. 5 and 6 represent the flooded energy released by the stock market. In some case, the corresponding magnitudes of such points are much more intensive than the other ones having energies lower than $\bar{E}_f$ (The black points).

**Fig. 5**: The daily magnitude of shocks. $M_e = 0$ and $M_f = 1$ correspond to the magnitudes of the thresholds $\bar{E}_e$ and $\bar{E}_f$, respectively.

**Fig. 6**: The daily magnitude-energy relation

In the following, the aggregated magnitude of any financial crash ($AM$) can be calculated by

$$AM = \sum_{t=1}^{m} M_t$$

(15)

where $m$ is the number of business days during a financial crash [4]. If the value of $AM$ is non–
positive, the financial crisis has not occurred and if the value of $AM$ is positive, the financial crash has happened. For computing Eq. (15), the duration of crisis should be computed. The crisis duration is determined by considering all business days in the financial crisis. A business day is specified as the day where the value of energy is greater than the threshold energy. The result of the calculations is shown in Table 6.

### Table 6: The magnitude of the financial earthquake

<table>
<thead>
<tr>
<th>Earthquake</th>
<th>$M_1$</th>
<th>Rank</th>
</tr>
</thead>
<tbody>
<tr>
<td>The effect of the global market events in 1998</td>
<td>0.689</td>
<td>13</td>
</tr>
<tr>
<td>The effect of the beginning of the Second Gulf War in 2003</td>
<td>2.390</td>
<td>10</td>
</tr>
<tr>
<td>The effect of the first privatization in the stock exchange in 2003</td>
<td>8.335</td>
<td>1</td>
</tr>
<tr>
<td>The effect of the global financial crisis in 2007</td>
<td>6.399</td>
<td>3</td>
</tr>
<tr>
<td>The effect of the Edalat share brokerage liquidation in 2009</td>
<td>3.738</td>
<td>8</td>
</tr>
<tr>
<td>The effect of the devaluation of the national currency in 2012</td>
<td>6.290</td>
<td>4</td>
</tr>
<tr>
<td>The effect of the conflict effect in Syria in 2013</td>
<td>2.383</td>
<td>11</td>
</tr>
<tr>
<td>The effect of the first nuclear negotiations in 2013</td>
<td>3.211</td>
<td>9</td>
</tr>
<tr>
<td>The effects of ISIL's attack on Iraq and the decline in oil prices in 2014</td>
<td>4.146</td>
<td>6</td>
</tr>
<tr>
<td>The effect of the nuclear negotiations in 2015</td>
<td>3.897</td>
<td>7</td>
</tr>
<tr>
<td>The effect of the nuclear deal in Lausanne in 2015</td>
<td>0.802</td>
<td>12</td>
</tr>
<tr>
<td>The effect of the implementation of the nuclear deal in 2016</td>
<td>5.046</td>
<td>5</td>
</tr>
</tbody>
</table>

According to the results shown in Table 6, five prominent events that influenced heavily on Tehran exchange stock, respectively are: the first privatization impact of the stock exchange in 2003, United Nations Security Council's sanctions in 2010, global financial crisis in 2007, devaluation of the national currency in 2012 and the implementation of the nuclear deal between Iran and the Group5+1 in 2016.

### 5 Discussion and Conclusion

Economics and Seismology are two scientific disciplines that unlike the prevalent belief have the considerable similarities. In Seismology, it is discussed that an earthquake motion is happened by the release of stored potential energy into the kinetic energy of motion and this energy follows the power law. The same condition can be observed when a financial crisis occurs. To investigate the similarity between earthquake and crisis in present study, the kinetic energy released by Tehran Stock Exchange that was called the financial energy was obtained with the help of the daily data of the total trading volume and the total Tehran Stock price index from October 5, 1997 to September 5, 2016. In this regard, TP statistic, proposed by Pisarenko and et al [37], was used for estimating the critical point or the lower threshold, i.e. the point beyond that the market energy follows from the power law distribution. Therefore, the occurrence of earthquake in Tehran Stock Exchange as the first hypothesis was confirmed. The tail statistical behavior was complicated. The results indicated the deviation of the energy from the Pareto distribution in the high quantiles of the data. Hence, there was upper threshold that the energy's distribution was changed from the Pareto to another distribution. It was estimated by TP statistic. The energy value followed the Wakeby distribution from the second threshold on. A simulation study was also employed for checking out the statistical behavior of the estimated thresholds.
In addition, the magnitude of the financial earthquakes was calculated by combination of two probability distributions. The results indicate that the domestic and the international events have caused the financial earthquakes in Tehran Stock Exchange. In this study, five prominent events influenced heavily on Tehran exchange stock that, respectively, are: the first privatization impact in the stock exchange in 2003, United Nations Security Council's sanctions in 2010, global financial crisis in 2007, devaluation of the national currency in 2012 and the implementation of the nuclear deal between Iran and the Group 5+1 in 2016. According to results, the impact of the 2007-2009 global financial crisis on Tehran Stock Exchange as the second hypothesis was confirmed. The findings were consistent with the studies that have been generally conducted in the field of the transmission of the global financial crisis to countries. Also, there was positive relation between the daily financial energy and the daily magnitude of the shocks that was connected by Gutenberg and Richter [31]. Hence, three hypotheses in this study were confirmed. The results of this study contain useful information for financial analysts and policymakers. It is necessary that the preventive policy packages are considered by the government and politicians. The policy suggestion is the evaluation of the repetitive behavior of the financial earthquakes in the world economic level by domestic politicians.

References


An analysis of the repeated financial earthquakes


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An analysis of the repeated financial earthquakes


