Advances in mathematical finance & applications, 4 (3), (2019), 49-58
DOI: 10.22034/amfa.2019.1869742.1231

Published by IA University of Arak, Iran
Homepage: www.amfa.iau-arak.ac.ir

Confidence Interval for Solutions of the Black-Scholes Model

Mohammad Ali Jafari [a], ***Mehran Paziresh*** [*,a], ***Majid Feshari*** [b]

[a] *Department of Financial Sciences, Kharazmi University, Tehran, Iran*
[b] *Department of Economics, Kharazmi University, Tehran, Iran*

ARTICLE INFO

Article history:
Received 26 May 2019
Accepted 30 July 2019

Keywords:
The asset valuation models
Confidence interval
Black Scholes model
Stochastic Differential equations

ABSTRACT

Improving return forecasting is very important for both investors and researchers in financial markets. In this study we try to aim this object by two new methods. First, instead of using traditional variable, gold prices have been used as predictor and compare the results with Goyal's variables. Second, unlike previous researches new machine learning algorithm called Deep learning (DP) has been used to improve return forecasting and then compare the results with historical average methods as bench mark model and use Diebold and Mariano's and West's statistic (DMW) for statistical evaluation. Results indicate that the applied DP model has higher accuracy compared to historical average model. It also indicates that out of sample prediction improvement does not always depend on high input variables numbers. On the other hand, when using gold price as input variables, it is possible to improve this forecasting capability. Result also indicate that gold price has better accuracy than Goyal's variable to predicting out of sample return.

1 Introduction

Determining model to provide an adequate estimate of future stock values will be important for investors in financial markets [4,9,11], which we analysed in this paper Black-Scholes model, and computer simulations are presented. In this paper, real stock market data is applied, say, Mellat Bank and Ansar Bank. The reason for choosing these two stocks is that the monthly data for these two stocks has existed for one year, that is, each month the transaction was conducted for stocks (Sales and purchases). The volatility of financial markets has attracted investors and economic activists in recent decades, and because fluctuations in financial markets are high, a model than can predict stock values in the future in necessary. Continuous time models based on stochastic differential equations are applied more than a century. Louis Bachelier in Paris, when studying the dynamic behaviour of the Paris stock exchanges 1900, provided a model for moving stocks. That why many believe that they are the founders of financial mathematics, indeed the stock model was expressed as follows:

$$S_t = S_0 + \sigma W_t \qquad (1)$$

* **Corresponding author Tel.:** *+9809369251061*
E-mail address: mehranpazeresh@gmail.com

where W_t is a Brownian motion, σ stock volatility and S_0 stock primary price. Since in this model, S_t can be a negative, it is not applied now. Pavel Samuelsson has shown that the proper pattern for commodity price changes in the stock market is the geometric Brownian motion. This model is as follows:

$$S_t = S_0 \exp[\mu t + \sigma W_t] \tag{2}$$

That is, $\mathbf{lnS_t}$ it has a normal log distribution. It is proved by the help of the Ito differential calculus- which is in fact the solution of the differential equation $\frac{\mathbf{dS_t}}{\mathbf{S_t}} = \boldsymbol{\sigma}\mathbf{dW_t} + \left(\boldsymbol{\mu} + \frac{\boldsymbol{\sigma}^2}{2}\right)\mathbf{dt}$.
The question we are looking for in this article is to answer it. Whether or not the performance of a Black-Scholes model is suitable for valuing and predicting the value of the stock price? The present article is presented in five sections, thus after the introduction in the first part, the second introduces the studies carried out in this field. In the third section, we present general introduction to the classical model of asset valuation, such as the Black-Scholes model, and basic concepts including stochastic process, stochastic differential equations, confidence interval, calibrations, and the method used in this study, namely, the Euler maruyama numerical method, and the mathematical equations are expressed. In the fourth part of the computer simulation, on using data from Mellat Bank and Ansar Bank shares in the year 2017-2018, is expressed by the maple software. In the fifth section, the performance of the Black-Scholes model in the valuations stock price is evaluated, for in this job average of the answers obtained from the computer simulation, compared with exact answer.

2 Literature Review

The subject we present in this article is a new and applicable. Similar work in this area can be found in the paper by Steven Dumber in 2013 from the Department of mathematics at the university of Nebraska-Link on obtaining the confidence interval for European put option has been highlighted by the Monte Carlo method [12]. Finding the confidence interval for the options price by Monte Carlo simulation is an appropriate method using a random sample of a special distribution that produces a numerical solution for a mathematical model. The Monte Carlo method is a Statistical technique among various mathematical methods that establishes a connection between a mathematical model statistic. By considering Brownian motion as a random model for the price process, we can use the Monte Carlo process to simulate the price option. We assume that the price is modeled as a geometric Brownian motion, so we can obtain its normal distribution function at the time of expiration. N the pseudo random number $\{x_1, \dots \dots . x_n\}$, from the normal distribution of the stock price models, then we approximate the price of the sales option with the expected yield: $g(s) = \text{Max}(k - s, 0)$, the average sample:

$$E[g(s)] \cong \frac{1}{n}\sum_{i=1}^{n} g(x_i) = \bar{g}_n \tag{3}$$

Which $\bar{g}_n$ is a normal distribution with mean: $E[(g(s)]$ and Variance $\frac{var[g(s)]}{n}$ is, as a result, normal distribution:

$$\bar{g}_n \sim N\left(E[g(s)], \frac{var[g(s)]}{n} \right) \tag{4}$$

This expression states the value $\bar{g}_n$ estimated by the Monte Carlo simulation method has the expected yield: which: $E[(g(s)]$ is the standard deviation by $\sqrt{n}$ divisibility. Most carefully we remember the normal distribution of the standard $p[z < 1.96] \cong 0.95$ then by the normal distribution properties, we can construct an confidence interval with a confidence level of 0 .095 which is as follow:

$$\left(E[g(s)] - 1.96\sqrt{\frac{var[g(s)]}{n}}, E[g(s)] + 1.96\sqrt{\frac{var[g(s)]}{n}} \right) \tag{5}$$

William Harley from the department of mathematics and computer at the royal Military University of Canada in 2011 has been working under an accidental strike to obtain a confidence interval for discounted equity cash dividends [5]. In this paper, the method for discounting stock profits is a common method in most financial books, generally, the Gordon growth model is one of the techniques used to calculate the cost of equity capital. Gordon assumes that the trend of dividend yield increases daily with constant geometric growth rates, and it is assumed that the stock return on profits will follow a steady path in the future. The purpose of this paper is to build a confidence interval for discounted cash benefits under random variables (for furthers see [6,7]) but our main goal here is to obtain the confidence interval for solution of the Black-Scholes model.

3 Research Methodology

This paper is practical, the method has been used in the study is Euler-Marumayama numerical method, the method for collecting data is the software Tse Clint, and analysis data by using the maple software. In the following, we give general over view of the variables studied in this paper, such as the Black-Scholes model, the Stochastic differential equation, the Stochastic process, the Euler Maruyama numerical method, and the confidence interval, and give a definition of them.

3.1 The Asset Classical Model

In the early 1970, Fisher Black, Mirren Scholes, Robert Merton, took a great of deal option pricing. The result was a model, known as the Black-Scholes model [2]. This model has a great deal of influence on pricing and trading option. This also played a major role in the success of financial engineering in the 1980s and 1990s. The Black-Scholes model is also used in stock valuation. The Black-Scholes model follows a geometric Brownian motion, and has a differential form as follows:

$$ds_t = \mu s_t dt + \sigma s_t dwt \tag{6}$$

3.2 Stochastic Deferential Equation

The stochastic differential equation is a differential equation in which one or more variable are stochastic process. In this modeling, white noise is usually used as a completely randomized parameter [1]. This equation is generally written in the following form:

$$dX_t = f(X_t, t)dt + g(X_t, t)dw_t \tag{7}$$

3.3 Stochastic Process

$\{ B(t), t \geq 0\}$ Is called a standard Brownian motion (wiener process) [8,10]. Which has the following properties:

a. The beginning of the process is zero that is: $B(0) = 0$

b. For each, $t, s \geq 0$ as $0 \leq s \leq t$ random variable $B(t) - B(s)$ has a normal distribution with mean zero and variance $(t - s)$.

c. Has a Gaussian property, that is, for any $t_1, t_2, t_3, \dots t_n$ random variables $B(t_1), B(t_1), \dots B(t_n)$ have a normal distribution.

d. Random variables $B(t_n) - B(t_{n-1}), \dots, B(t_2) - B(t_1)$ for $t_1 \leq t_2 \leq \cdots t_n$ are independent.

e. All sample paths B with a probability are continuous.

3.4 Euler Maruyama Numerical Method

This numerical method has been used for computer programming in the next section by the maple software, therefore, in this section, we describe the Euler Murayama discretization method is one of the methods for the discretization of the results. The method is equivalent to approximating integrals using the left end point rule when is $\mu(x_t; \theta)$ at timet_i . Hence integral are approximated as [3]

$$\int_{t_i}^{t_{i+1}} \mu(x_u; \theta) du \simeq \mu(\theta_{t_i}; \theta) \int_{t_i}^{t_{i+1}} du = \mu(x_t; \theta)\Delta, \tag{8}$$

The second part is approximated as follows:

$$\int_{t_i}^{t_{i+1}} \sigma(x_u; \theta) dw_u \simeq \sigma(x_{t_i}; \theta) \int_{t_i}^{t_{i+1}} dw_u = \sigma(x_{t_i}; \theta)(w_{t_{i+1}} - w_{t_i}) = \sigma(x_{t_i}; \theta)\sqrt{\Delta}\, z \tag{9}$$

The Z random variable is standard normal, hence the discretization is obtained as follows:

$$x_{t_{i+1}} = x_{t_i} + \mu(x_{t_i}; \theta)\Delta + \sigma(x_{t_i}; \theta)\sqrt{\Delta} z \tag{10}$$

3.5 Confidence Interval

The Confidence interval is divided into two-way confidence intervals. The normal distribution range is as follows: $x \sim N(\mu, \frac{\sigma^2}{n})$, so if we $z = \frac{x-\mu}{\frac{\sigma}{\sqrt{n}}}$ define, Z normal will be standardized. Therefore: $p\{-z < z < z\} = 1 - \alpha$ will be. So the two way confidence interval $1 - \alpha$ for the known variance state is a follows:

$$\mu - \frac{\sigma}{\sqrt{n}} z_{1-\frac{\alpha}{2}} < x < \mu + \frac{\sigma}{\sqrt{n}} z_{1-\frac{\alpha}{2}} \tag{11}$$

Also, the one-way confidence interval is dividend in two groups of on-sided left and right sides from the right, sometimes it is important for us to know that x is more or less than a certain number, or if the image is less than a certain amount, then
one-way confidence interval we use, as a result:

$$p\{z < T\} = 1 - \alpha \tag{12}$$

So: $z = z_{1-\alpha}$ as a result:

$$p\left\{\frac{x-\mu}{\frac{\sigma}{\sqrt{n}}} < z_{1-\alpha}\right\} = 1 - \alpha \tag{13}$$

So the one-way confidence interval $1 - \alpha$ from the right is:

$$p\left\{x > \mu - \frac{\sigma}{\sqrt{n}} z_{1-\alpha}\right\} = 1 - \alpha \tag{14}$$

And one-way confidence interval on the left are the following:

$$p\left\{x < \mu + \frac{\sigma}{\sqrt{n}} z_{1-\alpha}\right\} = 1 - \alpha \tag{15}$$

4 Research Results

In this section, we describe, the discretization of the Black-Scholes model in with the Euler-Maruyama method, and then, using the Ansar bank and Mellat bank stocks data, and implementing with the maple software, will evaluate the performance of the Black-Scholes model in the valuation stock price, we checking do this with the average of the computer simulations and compare it with actual answers.

4.1 Implementation of the Black-Scholes Model

Consider the following Black-Scholes model:

$$\mathrm{dS_t} = \mathrm{\mu S_t dt} + \mathrm{\sigma S_t dW_t} \tag{16}$$

Which μ is the average return on the stock price, S_t the stock price at the moment t, σ and the standard deviation of the stock price.

The discretization of the Black-Scholes model with the maximum likelihood of Euler Murayama in the interval $[t_{i-1}, t_i]$ is as follows:

$$\int_{t_{i-1}}^{t_i} dS_t = \int_{t_{i-1}}^{t_i} \mu S_t dt + \int_{t_{i-1}}^{t_i} \sigma S_t dW_t \tag{17}$$

Therefore, considering the numerical approximation $\int_a^b f(t)dt = f(a)(b-a)$, we have:

$$S_{t_i} - S_{t_{i-1}} = \mu S_{t_{i-1}} + \sigma S_{t_{i-1}}\left(W_{t_i} - W_{t_{i-1}}\right) \tag{18}$$

The $(W_{t_i} - W_{t_{i-1}}) \sim N(0,h)$ random number is normal, therefore we have:

$$S_{t_i} = (1+\mu)S_{t_{i-1}} + \sigma S_{t_{i-1}} N(0,h) \tag{19}$$

4.2 Implementation of the Black-Scholes Model Which Real Data

Here we implement the Black-Scholes model with real stock data. We get data through the Tse Clint software.

For example, the monthly data of the mellat Bank index from 24/1/2017 to 23/1/2018 are as follows:

Table 1: Monthly data of Mellat Bank shares in 2017-2018

month	Symbol	Date	price	Returns	Logarithmic Returns
0	webmellat	24/01/2017	1206		
1	webmellat	25/02/2017	1099	-0.08872	-0.09291
2	webmellat	25/03/2017	1105	0.00546	0.005445
3	webmellat	24/04/2017	1008	-0.08778	-0.09188
4	webmellat	24/05/2017	1049	0.040675	0.039869
5	webmellat	24/06/2017	934	-0.10963	-0.11612
6	webmellat	18/07/2017	1025	0.09743	0.092971
7	webmellat	23/08/2017	1001	-0.02341	-0.02369
8	webmellat	23/09/2017	971	-0.02997	-0.03043
9	webmellat	23/10/2017	943	-0.02884	-0.02926
10	webmellat	22/11/2017	997	0.057264	0.055684
11	webmellat	23/12/2017	1111	0.114343	0.108265
12	webmellat	23/01/2018	1058	-0.0477	-0.04888

In table 1, the monthly price index, Mellat shares, returns and continuous compound returns (logarithmic returns) are presented. In the next step, we obtain the average returns on stock price and the Standard deviation of the stock, so the average returns and Standard deviation are equal to:

Mean = -0.010911333333333

Standard Deviation = 0.2535854534

Now given that the index's initial rate in Table1 is initially $S_0 = 1206$, we estimate the price for (one year later) S_1, by Euler Maruyama numerical method. Using the Euler Maruyama discretization and programming in the maple software, we can plot the normal histogram for the data as follows: (the number of simulation of the program: 1000)

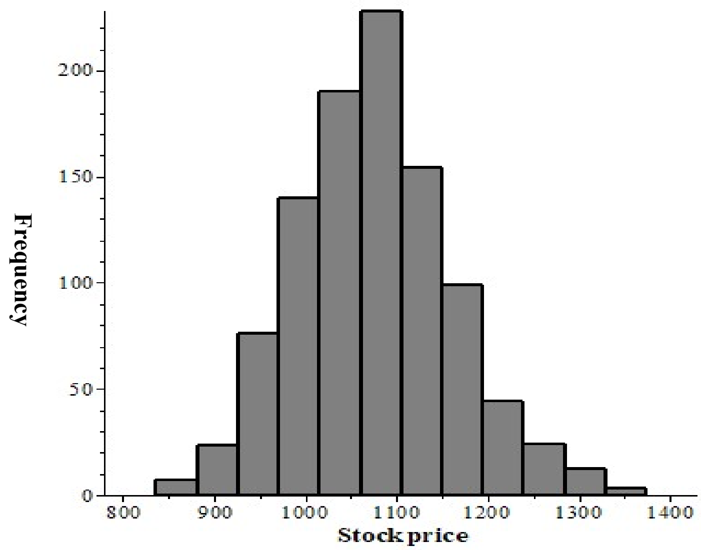

Fig. 1: Simulation of Mellat Bank Stock Data with Black-Scholes Model

Table 2: Monthly data of Ansar Bank shares in 2017-2018

month	Symbol	Date	price	Returns	Logarithmic Returns
0	weAnsar	25/03/2017	2174		
1	weAnsar	24/04/2017	2198	0.01104	0.010979
2	weAnsar	24/05/2017	2257	0.026843	0.026489
3	weAnsar	25/06/2017	2281	0.010634	0.010577
4	weAnsar	26/07/2017	2130	-0.0662	-0.06849
5	weAnsar	26/08/2017	2064	-0.03099	-0.03148
6	weAnsar	25/09/2017	1974	-0.0436	-0.04458
7	weAnsar	25/10/2017	1972	-0.00101	-0.00101
8	weAnsar	25/11/2017	2060	0.044625	0.043658
9	weAnsar	25/12/2017	2113	0.025728	0.025403
10	weAnsar	24/01/2018	2101	-0.00568	-0.0057
11	weAnsar	25/02/2018	2096	-0.00238	-0.00238
12	weAnsar	25/03/2018	2140	0.020992	0.020775

The average returns, Standard deviation and stock confidence interval for the simulated data is obtained as follows:

Mean =1075.17624527480
Standard Deviations =84.98752187
Confidence Interval = (908.6007021, 1241.751788)

Due to the average of the computer simulations and stock prices in the next one year, S_1 which according to Table 1 is equal to 1058, which is close to our prediction**.** In the next section, we will simulated a computer for Ansar Bank shares. As before, we receive data from the Tse Clint software. Ansar Bank's monthly stock data from 25/03/2017 to 25/03/2018 is as follows.

In table 2, the monthly price index of Ansar Bank shares, the continuous compound returns (logarithmic returns), are presented. In the next step, we obtain the average return on the stock price and the annual Standard deviation of the stock, thus the average annual Standard deviation is equal to:

Mean = -0.00131325000000000

Standard Deviation =0.1125153972

Now, given that the index's initial rate is initially, $S_0 = 2174$,we estimate the price for (one year later), S_1, using the method described in the previous section, on Euler Maruyama discretization and programming in the maple software, we can normal histogram for the data to draw the following: (the number of simulation of the program:1000)

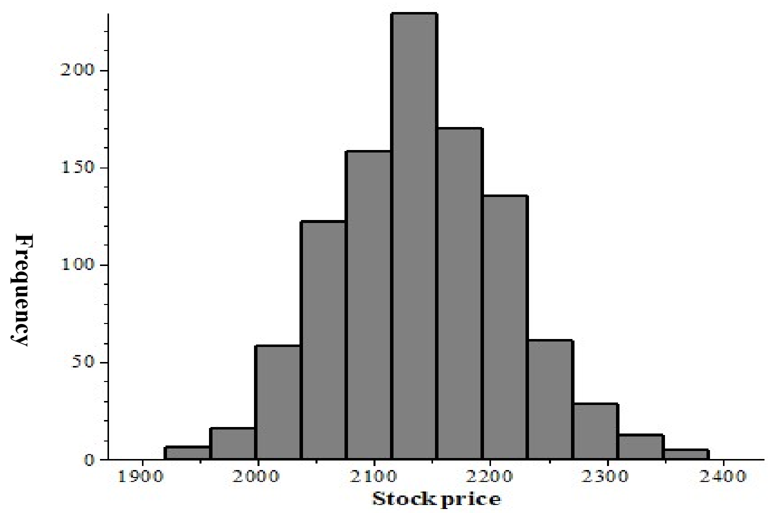

Fig. 2: Simulation of Ansar Bank Stock Data with Black-Scholes Model

The average return, Standard deviation and stock confidence interval for the simulated data is obtained as follows:

Mean = 2140.84270800000

Standard Deviations = 74.21274397

Confidence Interval = (1995.385730, 2286.299686)

Due to the average of the computer simulations and stock price one year later, $S_1 = 2140$ our prediction is accurate.

5 Conclusion

In this forth section, we implemented the Black-Scholes model based on stock data including the shares of Mellat Bank and Ansar Bank in the maple software, which yielded the following:

- Initial value of Mellat Bank shares in table1: $S_0 = 1206$

- The value one year after the stock of the Mellat Bank shares in table1: $S_1 = 1058$
- The average of computer simulation answers on the maple software: $S_1 = 1075$

The following results were also obtained for the shares of Ansar Bank:

- Initial value of Ansar Bank shares in table2: $S_0 = 2174$
- The value one year after the stock of the Ansar Bank shares in table2: $S_1 = 2140$
- The average of computer simulation answers on the maple software: $S_1 = 2140$

Based on the average of the answers obtained from computer simulations in the maple software, the performance of the Black-Scholes model in predicting stock value is quite appropriate, the shares intended for the computer implementation are entirely optional, but the stock has been tried to use, with their monthly data available for one year. That is, stocks have been sold in these months.

We have only used the Euler Maruyama method in this article. Undoubtedly, using more powerful numerical methods such as Milstein and Runge-kutta can achieve beter results. We also use wiener noise, which cannot model jump phenomena undoubtedly, the use of noise Levy can obtain better results.

In this field, we can refer to the article by Alireza kazeroni, pouyan kiani, and Zana mozaffari on estimating interest rate in Iran using fuzzy logic. This estimation interest rate on using fuzzy logic can be the basis for effective monetary and currency policies to target monetary and financial problems. The difference in our work with this article is that there are no other factors affecting stock price estimation, as well as we use the Euler Maruyama numerical method and programming in the maple software. But as mentioned, fuzzy logic is used in this paper, and other factors, such as inflation rate, housing efficiency, and liquidity volume in fluency the estimation of interest rates. Also, fuzzy logic does not require complicated mathematical modeling, but we use the Black-Scholes model to predict stock values. These models are suitable for short time periods, because the white noise has not memory.

References

[1] Bjork, T., *Arbitrage theory in continuous time*, Oxford university press, 2009.

[2] Black, F., Scholes, M., *The pricing of options and corporate liabilities*. Journal of political economy, 1973, **81**(3), P. 637-654.

[3] Cyganowski, S., Kloeden, P., Ombach, J., *From Elementary Probability to Stochastic Differential Equations with MAPLE*. Springer, 2001.

[4] Ehteshami, S., Hamidian, M., Hajiha, Z., Shokrollahi, S., *Forecasting Stock Trend by Data Mining Algorithm*, Advances in mathematical finance & applications, 2018, **3**(1), P.97-105. Doi: 10.22034/AMFA.2018.539138

[5] Gordon, M. J., *The investment, financing, and valuation of the corporation*. RD Irwin, 1962.

[6] Hull, J. C., Basu, S., *Options, futures, and other derivatives*. Pearson Education India, 2016.

[7] Hull, J., Treepongkaruna, S., Colwell, D., Heaney, R., Pitt, D., *Fundamentals. Of futures and options markets*. Pearson Higher Education AU, 2013.

[8] Klebaner, F. C., *Introduction to stochastic calculus with applications*. World Scientific Publishing Company, 2005.

[9] Nasr, N., Farhadi Sartangi, M., Madahi, Z., *A Fuzzy Random Walk Technique to Forecasting Volatility of Iran Stock Exchange Index*, Advances in mathematical finance & applications, 2019, **4**(1), P.15-30. Doi: :10.22034/AMFA.2019.583911.1172.

[10] Øksendal, B., *Stochastic differential equations: An Introduction with Applications*, Springer, 2003.

[11] Rezaei, N., Elmi, Z., *Behavioural Finance Models and Behavioural Biases in Stock Price Forecasting,* Advances in mathematical finance & applications, 2018, **3**(4), P.67-82. Doi:10.22034/AMFA.2019.576127.1118.

[12] Ross, S., *Simulation*, Academic Press, 2013.